KANNUR UNIVERSITY

(Abstract)

M.Sc. Mathematics programme– 2014 admission- Anomalies in the Syllabus of M.Sc. Mathematics Programme -Under Credit Based Semester System in Affiliated Colleges- rectified- modified – orders issued.

ACADEMIC BRANCH

U.O. No. Acad/C4/7538/2014 Dated, Civil Station (P.O), 07-08-2015

Read:-1.U.O No. Acad/C4/7538/2014 dated 24-07-2014

2.Minutes of the meeting of the Board of Studies in Mathematics (PG) held on 25-06-2015

3. Letter dated 29-06-2015 from the Chairman, Board of Studies in Mathematics (PG)

ORDER

1. As per the paper read (1) above the Scheme, Syllabus and Model Question Papers of M.Sc. Mathematics Programme Under Credit Based Semester System in affiliated colleges were implemented in the University w.e.f.2014 admission.

2. As per the paper read (2) above the meeting of the Board of Studies in Mathematics (PG) held on 25.06.2015 recommended to rectify the anomalies noticed in the implemented syllabus of M.Sc. Mathematics programme. The Board of Studies recommended to delete some portions of the papers **I Sem. MAT 1 C 04, II Sem. MAT 2 C 08 & II Sem. MAT 2 C 10 w. e. f. 2015 admission and the changes to be applicable for supplementary/ improvement examination of 2014 admission also.**

3. As per the paper read (3) above, the Chairman, Board of Studies in Mathematics (PG) forwarded the modified Syllabus after rectification of anomalies and for implementation.

The following portions are deleted from the above three papers

MAT 1 C04 - BASIC TOPOLOGY

Unit II Sections 2.5 and 2.6 of chapter 2 deleted

MAT 2 C08 - TOPOLOGY

Unit I Section 5.6 of chapter 5 deleted
Unit III Sections 8.3 and 8.4 of chapter 8 deleted.

MAT 2 C10 PARTIAL DIFFERENTIAL EQUATIONS AND INTEGRAL EQUATIONS

Unit I Sections 1.11 of chapter 1 deleted.
Unit II Sections 2.4.6 to 2.4.13 and sections 2.7 and 2.8 deleted.
Unit III Sections 3.10 and 3.11 deleted.

5. The Vice-Chancellor, after considering the matter in detail, and in exercise of the powers of the Academic Council, conferred under Section 11 (1) of Kannur University Act, 1996 and all other enabling provisions read together with, has accorded sanction to implement the modified Syllabus and Model Question papers (Core Courses) for M.Sc. mathematics Programme in affiliated Colleges under Credit Based Semester System subject to ratification by the Academic Council.

P.T.O

6. The order as per paper read (1) stands modified in respect of the above three papers to this effect.

7. Orders are, therefore, issued accordingly.

8. The modified pages of Syllabus and Model Question papers are appended accordingly

Sd/-
JOINT REGISTRAR (ACADEMIC)
For REGISTAR

To:

The Principals of Colleges offering M.Sc. Mathematics Programme.

Forwarded /By Order

SECTION OFFICER

Copy to:

1. The Examination Branch (through PA to CE).
2. The Chairman, BOS in Mathematics (PG).
3. PS to VC/PA to PVC
4. PA to Registrar/PA to CE.
5. JR/AR-I (Academic).
6. Computer Programmer (for uploading in the website)
7. SF/DF/FC

KANNUR UNIVERSITY CIVIL STATION P.O. KANNUR Pin-670 002 THAVAKKARA

- ***For more details log on to www. Kannur university.ac.in***

M.Sc. Degree (Mathematics)

MAT1C04 : BASIC TOPOLOGY

Text: C. Wayne Patty, *Foundations of Topology*, Second Edition – Jones & Bartlett India Pvt. Ltd., New Delhi, 2012.

Unit – I

Topological Spaces: The Definition and Examples, Basis for a Topology, Closed Sets, Closures and Interiors of Sets, Metric spaces, Convergence, Continuous functions and Homeomorphisms.

[Chapter 1 Sections 1.2 to 1.7]

Unit – II

New spaces from old ones: Subspaces, The Product Topology on X x Y, The Product Topology, The Weak Topology and the Product Topology.

[Chapter 2 Sections 2.1 to 2.4]

Unit – III

Connectedness and Compactness: Connected spaces, Pathwise and local connectedness, Totally disconnected space, Compactness in metric spaces, Compact spaces.

[Chapter 3 all sections, Chapter 4 Sections 4.1 and 4.2]

References:

1. K. D. Joshi, *Introduction to General Topology*, New Age International (P) Ltd., Publishers.
2. Dugundji, *Topology*, Prentice Hall of India.
3. G. F. Simmons, *Introduction to Topology and Modern Analysis*, Mc Graw Hill.
4. S. Willard, *General Topology* , Addison Wesley Publishing Company.
5. J. R. Munkres, *Topology: A First Course*, Prentice Hall of India.
6. Murdeshwar M. G., *General Topology*, second edition, Wiley Eastern.
7. Kelley, *General Topology*, van Nostrand, Eastern Economy Edition.

M.Sc. Degree (Mathematics)

MAT2C08 : TOPOLOGY

Text: C. Wayne Patty, *Foundations of Topology*, Second Edition – Jones & Bartlett India Pvt. Ltd., New Delhi, 2012.

Unit – I

The Separation and Countability Axioms: T_0, T_1 & T_2 spaces, Regular and completely regular spaces, Normal and completely normal spaces, The countability axioms, Urysohn's Lemma and Tietze Extension Theorem.

[Chapter 5 Sections 5.1 to 5.5]

Unit – II

Compactness: Local compactness and the relation between various forms of compactness, The weak topology on a topological space. Equicontinuity; Compactifications, The Alexander Subbase and Tychonoff Theorems, Urysohn's Metrization Theorem,

[Chapter 4 Sections 4.3 to 4.5; Chapter 6 Sections 6.6 and 6.7; Chapter 7 Section 7.1]

Unit – III

Homotopy of Paths - The Fundamental Group.

Chapter 8 Section 8.1 to 8.2]

References:

1. K. D. Joshi, *Introduction to General Topology*, New Age International (P) Ltd., Publishers.
2. Dugundji, *Topology*, Prentice Hall of India.
3. G. F. Simmons, *Introduction to Topology and Modern Analysis*, Mc Graw Hill.
4. S. Willard, *General Topology* , Addison Wesley Publishing Company.
5. J. R. Munkres, *Topology: A First Course*, Prentice Hall of India.
6. Murdeshwar M. G., *General Topology*, second edition, Wiley Eastern.
7. Kelley, *General Topology*, van Nostrand, Eastern Economy Edition.

MAT2C10:PARTIALDIFFERENTIALEQUATIONSANDINTEGRAL EQUATIONS

Text Book: 1.AmarnathM: PartialDifferentialEquations,Narosa,NewDelhi(1997)
2.HildebranF.B: MethodsofAppliedMathematics,(2nd Edition)Prentice-HallofIndia,NewDelhi(1972)

UNIT I

First Order P.D.E.

Curves and Surfaces, Genesis of first order Partial Differential Equations, Classification of integrals, Linear equations of first order, Pfaffian differential equations, Compatible systems, Charpit's method, Jacobi's method, Integral surfaces passing through a given curve, Quasi linear equations.
[Sections 1.1 – 1.10. from the Text 1]

UNIT II

Second Order P.D.E.

Genesis of second order Partial Differential Equations.
Classification of second order Partial Differential Equations.
One dimensional Wave Equation:

Vibrations of an infinite String , Vibrations of semi-infinite String, Vibrations of a String of Finite Length, Riemann's Method, Vibrations of a String of Finite Length (Method of Separation of Variables).

Laplace's Equation:

Boundary Value Problems, Maximum and Minimum Principles, The Cauchy Problem, The Dirchlet Problem for the Upper Half Plane,The Neumann Problem for the Upper Half Plane.

Heat Conduction Problem:
Heat Conduction - Infinite Rod Case, Heat Conduction – Finite Rod Case.
Duhamel's Principle:
Wave Equation, Heat Conduction Equation.
[Sections 2.1 – 2.6. from the Text 1. Omit sections 2.4.6 to 2.4.13]

UNIT III

Integral Equations.

Introduction ,Relation Between differential and Integral Equation, The Green's Function, Frdholm Equation With Separable Kernels, Illustrative Examples, Hilbert Schmidt Theory, Iterative Methods for Solving Equations of the Second Kind.
[Sections 3.1 – 3.3, 3.6 – 3.9 from the Text 2]

REFERENCES

1. E.A. Coddington : An Introduction to Ordinary Differential Equations
Printice Hall of India ,New Delhi (1974)
2. F. John : Partial Differential Equations
Narosa Pub. House New Delhi (1986)
3. Phoolan Prasad & : Partial Differential Equations
RenukaRavindran Wiley Eastern Ltd New Delhi (1985)
4. R. Courant and D.Hilbert : Methods of Mathematical Physics , Vol I
Wiley Eastern Reprint (1975)
5. W.E. Boyce & R.C. Deprima : Elementary Differential Equations and Boundary Value Problems
John Wiley & Sons, NY, 9th Edition
6. Ian Sneddon : Elements of Partial Differential Equations
McGraw-Hill International Edn., (1957)

UNIVERSITY MODEL QUESTION PAPER

KANNUR UNIVERSITY

FIRST SEMESTER M.Sc DEGREE EXAMINATION, NOVEMBER ….

Mathematics

MAT1C04: BASIC TOPOLOGY

Time: Three Hours | Maximum : 60 Marks

Part A

Answer **four** *questions from this part.*
Each question carries **3** *marks.*

1. Give six topologies on the set $\{1, 2, 3\}$.
2. Prove that in a metric space every convergent sequence is a Cauchy sequence. Is the converse true? Justify your answer with an example.
3. Let $X = \{1, 2, 3\}$, $\tau = \{\emptyset, \{1\}, \{1, 2\}, X\}$, $Y = \{4, 5\}$, and $U = \{\emptyset, \{4\}, Y\}$. Give a basis for the product topology on $X \times Y$.
4. For each natural number *n*, let $X_n = R$ and let τ_n be the discrete topology on X_n. Let τ be the product topology on $X = \prod_{n \in N} X_n$. Is τ the discrete topology on X? Either prove that it is or show by example that it is not.
5. Let (X, τ) be a topological space and define a relation $\sim$ on X by $x \sim y$ provided there is a pathwise connected subset A of X such that $x, y \in A$. Prove that $\sim$ is an equivalence relation on X.
6. Let A be a subset of a topological space (X, τ). Prove that A is compact if and only if every cover of A by members of τ_A has a finite subcover.

Part B

Answer **four** *questions from this part without omitting any Unit.*
Each question carries **12** *marks.*

UNIT I

7. (*a*) Prove that the topology generated by the square metric on R^2 is the usual topology.
   (*b*) State and prove the necessary and sufficient condition for a subset of the power set $P(X)$ to be a basis for a topology on X.

8. (*a*) Prove that every metric space is first countable.

(*b*) Let τ be the usual topology on R. Prove that $B = \{(a, b): a < b$ and a and b are rational$\}$ is a countable basis for τ .

9. (*a*) Prove that a subset A of a topological space (X, τ) is a perfect set if and only if it is closed and has no isolated points.

(*b*) Prove that metrizability is a topological property.

UNIT I I

10. (*a*) Prove that the topological properties Hausdorff and first countable are hereditary.

(*b*) Prove that every subspace of a separable metric space is separable.

11. (*a*) Let (X, τ), (Y_1, U_1) and (Y_2, U_2) be topological spaces, let $f_1 : X \to Y_1$ and $f_2 : X \to Y_2$ be functions, and define $f : X \to Y_1 \times Y_2$ by $f(x) = (f_1(x), f_2(x))$. The show that f is continuous if and only if f_1 and f_2 are continuous.

(*b*) Let $\{(X_\alpha, \tau_\alpha): \alpha \in \Lambda\}$ be an indexed family of topological spaces, and for each $\alpha \in \Lambda$, let B_α be a basis for τ_α. Then prove that the collection B of all sets of the form $\prod_{\alpha\in\Lambda} B_\alpha$, where $B_\alpha = X_\alpha$ for all but a finite number of members $\beta_1, \beta_2, \ldots, \beta_n$ of Λ and $B_{\beta_i} \in B_{\beta_i}$ for each $i = 1, 2, \ldots, n$ is a basis for the product topology τ on $\prod_{\alpha\in\Lambda} X_\alpha$.

12. (*a*) Let $\{(X_\alpha, \tau_\alpha): \alpha \in \Lambda\}$ be an indexed family topological spaces, and for each $\alpha \in \Lambda$, let $(A_\alpha, \tau_{A_\alpha})$ be a subspace of (X_α, τ_α). Then the product topology on $\prod_{\alpha\in\Lambda} A_\alpha$ is the same as the subspace topology on $\prod_{\alpha\in\Lambda} A_\alpha$ determined by the product topology on $\prod_{\alpha\in\Lambda} X_\alpha$.

(*b*) Let $\{(X_\alpha, \tau_\alpha): \alpha \in \Lambda\}$ be an indexed family of first countable spaces, and let $X = \prod_{\alpha\in\Lambda} X_\alpha$. Then (X, τ) is first countable if and only if τ_α is the trivial topology for all but a countable number of α.

UNIT I I I

13. (*a*) If X is an infinite set and τ is the discrete topology on X, then prove that (X, τ) is not compact.

(*b*) Prove that every totally bounded metric space is bounded.

14. (*a*) Prove that every countable compact space has the Bolzano-Weierstrass property.

(*b*) Prove that a metric space is compact if and only if it is closed and bounded.

15. (*a*) Prove that compactness is a topological property.

(*b*) Let (X, τ) be a compact space and let $f: X \to R$ be a continuous function. Then prove that there exist $c, d \in X$ such that for all $x \in X$, $f(c) \leq f(x) \leq f(d)$.

UNIVERSITY MODEL QUESTION PAPER

KANNUR UNIVERSITY

SECOND SEMESTER M.Sc DEGREE EXAMINATION, APRIL ….

Mathematics

MAT2C08: T O P O L O G Y

Time: Three Hours | Maximum : 60 Marks

Part A

Answer **four** *questions from this part.*
Each question carries **3** *marks.*

1. Prove that every subspace of a T_2 space is a T_2 space.
2. Give an example of a space that is normal but not regular.
3. Let (X, τ) be a topological space, and p be an object that does not belong to X, and let $Y = X \cup \{p\}$. Prove that $U = \{U \in P(Y) : U \in \tau$ or $Y - U$ is a closed compact subspace of $X\}$ is a topology on Y.
4. For each natural number n, let (X_n, d_n) be a metric space, let $X = \prod_{n \in N} X_n$. Define
$$d(x, y) = \sum_{n=1}^{\infty} \frac{d_n(x_n, y_n)}{2^n}$$
for all $x = (x_1, x_2, \ldots, x_n),\ y = (y_1, y_2, \ldots, y_n) \in X$. Prove that d is a metric on X.
5. Let (X, τ) be a topological space and let $x_0 \in X$. Prove that $\square_p$ is an equivalence relation on $\Omega(X, x_0)$.
6. Explain the terms covering map and covering space.

Part B

Answer **four** *questions from this part without omitting any Unit.*
Each question carries **12** *marks.*

UNIT I

7. (*a*) Let (X, τ) be a topological space. Then prove that the following statements are equivalent.
   (i) (X, τ) is a T_1 space.
   (ii) For each $x \in X$, $\{x\})$ is closed.
   (iii) If A is any subset of X, then $A = \bigcap \{U \in \tau : A \subseteq U\}$.

(*b*) Prove the result: A T_1 space (X, τ) is regular if and only if for each member p of X and each neighborhood U of p, there is a neighborhood V of p such that $\bar{V} \subseteq U$.

8. (*a*) Prove that every compact Hausdorff space is normal.

   (*b*) Let C be a closed subset of a normal space (X, τ). Then prove that (C, τ_C) is normal.

9. (a) Prove that every regular Lindelof space is normal.

   (b) State and prove Urysohn's lemma.

UNIT I I

10. (*a*) Prove that closed subspace of a locally compact Hausdorff space is locally compact.

    (*b*) Let (X, τ) and (Y, U) be topological spaces and let F be a finite set of conintuous functions that map X into Y. Prove that F is equicontinuous.

11. (*a*) Prove that every completely regular space has a compactification.

    (*b*) Prove that the product of compact spaces is compact.

12. State and prove Urysohn's metrization theorem.

UNIT I I I

13. (*a*) Let (X, τ) be a topological space, and let $x_0 \in X$. Furthermore, let $\alpha_1, \alpha_2, \beta_1, \beta_2 \in \Omega(X, x_0)$ and suppose $\alpha_1 \simeq_p \alpha_2$ and $\beta_1 \simeq_p \beta_2$. Then prove that $\alpha_1 * \beta_1 \simeq_p \alpha_2 * \beta_2$.

    (*b*) Let (X, τ) be a topological space, and let $x_0 \in X$, and let $[\alpha], [\beta], [\gamma] \in \pi_1(X, x_0)$. Then prove that $([\alpha] \circ [\beta]) \circ [\gamma] = [\alpha] \circ ([\beta] \circ [\gamma])$.

14. (*a*) Let (X, τ) be a topological space, and let $x_0 \in X$, and let $e: I \to X$ be the path defined by $e(x) = x_0$ for each $x \in I$. Then prove that $[\alpha] \circ [e]) = [e] \circ [\alpha]$ for each $[\alpha] \in \pi_1(X, x_0)$.

    (*b*) Let (X, τ) be a topological space, and let $x_0 \in X$, and let $[\alpha] \in \pi_1(X, x_0)$. Then prove that there exists $[\tilde{\alpha}] \in \pi_1(X, x_0)$ such that $[\alpha] \circ [\tilde{\alpha}] = [\tilde{\alpha}] \circ [\alpha] = [e]$.

15. (*a*) Let (X, τ) be a path wise connected space, and let $x_0, x_1 \in X$. Then prove that $\pi_1(X, x_0)$ is isomorphic to $\pi_1(X, x_1)$.

    (*b*) Let (X, τ) and (Y, U) be topological spaces, and let $x_0 \in X$ and $y_0 \in Y$. If (X, x_0) and (Y, y_0) are of the same homotopy type, then show that $\pi_1(X, x_0)$ is isomorphic to $\pi_1(Y, y_0)$.

MODEL QUESTION PAPER
KANNUR UNIVERSITY
SECOND SEMESTER M.Sc DEGREE EXAMINATION
MAT 2C10 PARTIAL DIFFERENTIAL EQUATIONS AND INTEGRAL EQUATIONS

Time 3 hours **Maximum Marks 75**

(PART A)

Answer any four questions from this part.
Each question carries 3 marks.

1. Eliminate the arbitrary function F from $F(x-z, y-z) = 0$ and find the corresponding Partial differential equation.
2. Find the Complete integral of the equation $zpq - p - q = 0$
3. Reduce the equation $4u_{xx} - 4u_{xy} + 5u_{yy} = 0$ to Canonical form.
4. Show that the solution of the Dirichlet problem, if it exist, is unique.
5. Show that the integral equation corresponding to boundary value problem, $\frac{d^2y}{dx^2} + \lambda y = 0,\ y(0) = 0,\ y(l) = 0$ is a Fredholm equation of the second kind.
6. Show that characteristic numbers of a Fredholm equation with real symmetric kernel are all real.

(PART B)

Answer Four questions from this part, without avoiding any unit.
Maximum Marks from this part is 48.

(UNIT I)

7. (a) Let $z = F(x, y, a)$ be a one parameter family of solutions of $f(x, y, z, p, q) = 0$. Show that this one parameter family, if it exists, is also a solution. (4)
   (b) Show that $(x-a)^2 + (y-b)^2 + z^2 = 1$ is a complete integral of $z^2(1 + p^2 + q^2) = 1$ (4)
   (c) Solve $x^2p + y^2q = (x + y)z$ (4)
8. (a) Obtain the necessary and sufficient condition of integrability of the Pfaffian differential equation $\overrightarrow{X}.\overrightarrow{dr} = P(x, y, z)dx + Q(x, y, z)dy + R(x, y, z)dz = 0$ (6)
   (b) Show that the necessary and sufficient condition for the integrability of $dz = \phi(x, y, z)dx + \psi(x, y, z)dy$ is $[f, g] = 0$ (6)
9. (a) Solve $u_x x^2 - u_y^2 - a u_z^2 = 0$ using Jacobi's Method (6)
   (b) Solve $xz_y - yz_x = z$, with the initial condition $z(x, 0) = f(x), x \geq 0$. (6)

(UNIT II)

10. (a) Derive d'Alembert's Solution of Wave Equation. (6)
    (b) Derive the Riemann function and hence obtain the solution of $u_{\xi\eta} = 0$ (6)
11. (a) State and Prove maximum Principle. (6)
    (b) Show that The solution of Neumann problem is unique up to the addition of a constant. (6)

12. (a) Solve the non - homogeneous wave equation $u_{tt} - c^2 u_{xx} = F(x,t),\ -\infty < x < \infty$ with homogeneous initial conditions $u(x,0) = u_t(x,0) = 0$ using Duhamel's Principle. (6)

(b) Solve $u_t = u_{xx}, 0 < x < l, t > 0, u(0,t) = u(l,t) = 0, u(x,0) = x(l-x), 0 \leq x \leq l.$ (6)

(UNIT III)

13. (a) Solve $y'' + xy = 1,\ y(0) = 0,\ y(l) = 1$ using Green's function. (6)

(b) If $y_m(x)$ and $y_n(x)$ are characteristic functions of $y(x) = \lambda \int_a^b K(x,\xi) y(\xi)\ d\xi$ corresponding to distinct characteristic numbers , then show that $y_m(x)$ and $y_n(x)$ are orthogonal over the interval (a,b) (6)

14. (a) Show that the non-homogenous Fredholm integral equation of second kind with separable kernel can be reduced to a system of linear algebraic equations and hence show that such equation may have a unique solution or an infinite number of solutions. (6)

(b) Solve the Fredholm integral Equation $y(x) = F(x) + \lambda \int_0^1 (1 - 3x\xi) y(\xi)\ d\xi$ (6)

15. (a) Describe the iterative method for Solving Fredholm equation of the Second Kind (6)

(b) Solve by iterative method $y(x) = 1 + \lambda \int_0^1 (1 - 3x\xi) y(\xi)\ d\xi$ (6)