KANNUR UNIVERSITY

(Abstract)

M.Sc. Mathematics Programme under Choice Based Credit Semester System in the University Department– Revised Scheme, Syllabus & Model Question Papers Implemented with effect from 2015 admission- Orders issued.

ACADEMIC 'C'SECTION

U.O. No.Acad/C4/ 6206/2015 Civil Station P.O, Dated, 30-10-2015

Read:
1. U.O No.Acad/C3/2049/2009 dated 11.10.2010.
2. U.O No.Acad/C3/2049/2009 dated 05.04.2011.
3. Meeting of the Syndicate Sub-Committee held on 16.01.2015.
4. Meeting of the Curriculum Committee held on 10.04.2015.
5. Meeting of the Department Council held on 16.04.2015.
6. U.O No.Acad/C4/14536/2014 dated 29.05.2015.
7. Letter from the HOD, Dept. of Mathematical Sciences, Mangattuparamba Campus
8. Meeting of the Curriculum Committee held on 03.09.2015.

ORDER

1.The Regulations for Post Graduate Programmes under Choice Based Credit Semester System were implemented in the Schools/Departments of the University with effect from 2010 admission as per the paper read (1) above and certain modifications were effected to the same vide paper read (2).

2. The meeting of the Syndicate Sub-Committee recommended to revise the Scheme and Syllabus of all the Post Graduate Programmes in the University Schools/Departments under Choice Based Credit Semester System (CCSS) with effect from 2015 admission vide paper read (3) above.

3. As per the paper read (4) above, the meeting of the Curriculum Committee recommended certain modifications/ additions to the Regulations for Post Graduate Programmes under Choice Based Credit Semester System and the Regulations were modified in the University w.e.f. 2015 admission vide paper read (6) above.

4. The Department Council vide paper read (5) above has approved the Scheme, Syllabus & Model Question Papers for M.Sc. Mathematics Programme under Choice Based Credit Semester System (CCSS) for implementation with effect from 2015 admission.

5. The HOD, Dept. of Mathematical Sciences vide paper read (7) above, has forwarded the Scheme, Syllabus & Model Question Papers for M.Sc. Mathematics Programme in line with the revised Regulations for Choice Based Credit Semester System for implementation with effect from 2015 admission.

P.T.O.

6. The meeting of the Curriculum Committee held on 03.09.2015 approved the Scheme; Syllabus & Model Question Papers for M.Sc. Mathematics Programme under Choice Based Credit Semester System in the Department vide paper read (8) above.

7. The Vice Chancellor after considering the matter in detail, and in exercise of the powers of the Academic Council conferred under section 11(1) of KU Act 1996, and all other enabling provisions read together with, has accorded sanction to implement the Scheme, Syllabus & Model Question Papers for M.Sc. Mathematics Programme under Choice Based Credit Semester System, offered in the University Department w.e.f 2015 admission, subject to report to the Academic Council.

8. Orders are, therefore, issued accordingly.

9. The revised Scheme, Syllabus and Model Question Papers effective from 2015 admission are appended.

JOINT REGISTRAR (ACADEMIC)
FOR REGISTRAR

To

The HOD, Department of Mathematical Sciences
Mangattuparamba Campus, Kannur University

Copy To:

1. The Examination Branch (through PA to CE)
2. PS to VC/PA to PVC/PA to R/PA to CE/PA to FO
3. JR/AR I Academic
4. The Computer Programmer *(for uploading in the website)*
5. SF/DF/FC

Forwarded/By Order

SECTION OFFICER

KANNUR UNIVERSITY CIVIL STATION P.O. KANNUR Pin-670 002 THAVAKKARA

For more details: log on www.kannur university .ac.in

KANNUR UNIVERSITY
DEPARTMENT OF MATHEMATICAL SCIENCES

Regulations, Scheme and Syllabus for
M.Sc. Mathematics Programme
With effect from 2015 admission

KANNUR UNIVERSITY
DEPARTMENT OF MATHEMATICAL SCIENCES

Regulations, Scheme and Syllabus for M.Sc. Mathematics Programme with effect from 2015 admission

1. **Eligibility for Admission**

The essential qualification for admission shall be a B.Sc. degree in Mathematics with at least 55% marks or equivalent CGPA in the core and complimentary (main and subsidiary)subjects together.

2. **Mode of Selection**

The selection of the candidates will be on the basis of the marks secured in the entrance test.

3.Mode of Instruction : English

4. **Course Structure**

The course is offered under **CCSS** with duration 2 years (4 semesters). Credit defines the quantum of contents/syllabus prescribed for a course and determines the number of hours of instruction required per week. There shall be at least sixteen week schedule per semester to complete the course contents.

The department council will assign an advisor(faculty member) to each of the student admitted. He/She advice the student about the academic programme and counsel on the choice of the course depending on the student's academic background and objective.

The department council shall prescribe the maximum number of students that can be admitted, taking into consideration of the facilities available. The minimum duration for the completion of the M.Sc. Mathematics course is four semesters and the maximum period for the completion is eight semesters.

No student shall register for more than 24 credits and less than 16 credits per semester.

The minimum total credits required for the successful completion of the M.Sc. programme is 80 and in which minimum credit required for the core course is 60 and the minimum for the elective is 12.

Those who secure only the minimum credit for core/elective subject has to supplement the deficiency required for obtaining the minimum total credits required for the successful completion of the programme from the other division

5. Evaluation

All the semesters will have continuous and end semester assessments. The course instructors carry out the internal assessment for each paper. For theory papers, the proportion of the distribution of marks among the continuous assessment and end semester examination shall be 40:60. Duration of the theory examination is 3 hours.

5.1 Continuous Evaluation

(i) Theory paper

Continuous assessment includes assignments, seminars, periodic written examinations and end semester viva-voce for each paper.

Weightage to the components of the continuous assessment shall be

Written test papers	:	40%	(16 marks)
Assignments	:	20%	(8 marks)
Seminar /Viva	:	40%	(16 marks)

(ii) Viva - Voce

For Viva - Voce, internal assessment (40 marks) will be awarded based on viva conducted by the faculty concerned.

5.2 End Semester Evaluation

Viva - Voce shall be conducted by two examiners (for each paper) one external and one internal. External examiner for Viva - Voce shall be selected from the list of experts provided by the Head of the Department / Chairman, BOE.

The end semester evaluation of each theory paper consist of written examination only (Maximum marks 60, 3 Hours duration)

For written examination, two questions shall be asked from each unit in the syllabus, of which the student is expected to answer any one and the marks for each question shall be 12

5.3 Project work

Each M.Sc. students has to carry out a research project during third and fourth semesters.

The project evaluation, comprising of internal (total 80 marks) and external (total 120 marks), will be carried out during fourth semester. The scheme of evaluation of project is as follows.

Total marks	:	200	
Content	:	30%	60 (36 external & 24 internal)
Methodology and Presentation	:	50%	100 (60 external & 40 internal)
Dissertation Viva-voce	:	20%	40(24 external & 16 internal)

5.4 Re-appearance of the end semester examination

A minimum grade point 5 is needed for the successful completion of a course. A student who has failed in a course can reappear for the end semester examination of the same course along with the next batch or choose another course in the subsequent semesters to acquire the minimum credits needed for the completion of the programme. A student who fails in any paper need to appear for re-examination in that paper only. There shall be no supplementary examination, No student shall allowed taking more than eight consecutive semesters for completing the programme from the date of enrollment.

SYLLABUS

The syllabus appended is applicable from 2015 admission.

M.Sc. Mathematics under CCSS
(2015 Admission)
Course Structure

Course Code	Course Title	Number of hours per week		Credits	CE	ESE	Total
		Lecture hours	Tutorial hours				
First Semester							
MAT C 101	Algebra I	4	2	4	40	60	100
MAT C 102	Linear Algebra	4	2	4	40	60	100
MAT C 103	Differential Equations I	4	2	4	40	60	100
MAT C 104	Real Analysis	4	2	4	40	60	100
MAT C 105	Topology	4	2	4	40	60	100
Total		**20**	**10**	**20**	**200**	**300**	**500**
Second Semester							
MAT C 201	Complex Analysis	4	2	3	40	60	100
MAT C 202	Functional Analysis	4	2	3	40	60	100
MAT C 203	Algebra II	4	2	3	40	60	100
MAT C 204	Differential Equations II	4	2	3	40	60	100
MAT C 205	Measure & Integration	4	2	3	40	60	100
MAT C 206	Viva-voce	-	-	5	40	60	100
Total		**20**	**10**	**20**	**240**	**360**	**600**
Third Semester							
MAT C 301	Differential Geometry	4	2	4	40	60	100
MAT C 302	Fuzzy Mathematics	4	2	4	40	60	100
MAT C 303	Probability	4	2	4	40	60	100
MAT E 304	Elective I	4	2	4	40	60	100
MAT C 305	Dissertation	-	6	-	-	-	-
Total		**16**	**14**	**16**	**160**	**240**	**400**
Fourth Semester							
MAT E 401	Elective 2	4	2	3	40	60	100
MAT E 402	Elective 3	4	2	3	40	60	100
MAT E 403	Elective 4	4	2	3	40	60	100
MAT E 404	Elective 5	4	2	3	40	60	100

MAT C 405	Dissertation	-	6	8	80	120	200
MAT C 406	Viva-voce	-	-	4	40	60	100
Total		**16**	**14**	**24**	**280**	**420**	**700**
Total							**2200**

List of Electives (to be added/chosen as per the requirements and availability of faculty)

1. Algebraic Geometry
2. Projective Geometry
3. Advanced Complex Analysis
4. Analytical Mechanics
5. Stochastic Processes
6. Fluid Mechanics
7. Algebraic Topology
8. Numerical Analysis and Computing
9. Graph Theory
10. Fractal Geometry
11. Coding Theory
12. Cryptography
13. Number Theory
14. Analytic Number Theory
15. Algebraic Number Theory
16. Advanced Functional Analysis
17. Operations Research

DETAILED SYLLABUS

MAT C 101 Algebra I

Unit 1
Direct products and finitely generated abelian groups. Homomorphisms. Factor groups. Factor group computations and simple groups. (Chapter 2 Section11 and Chapter 3 Sections 13-15 of Text 2.)
Unit 2
Group Action on a set,Application of G-sets to counting,Sylow theorems,Applications of the Sylow theory. (Chapter 3 Section16,17 and Chapter 7 Sections 36,37 of Text 2.)
Unit 3
Free abelian groups. Free groups. Group presentation. Field of quotients of the integral domain. Ring of polynomials. (Chapter 7 Sections 38-40, Chapter 4 Sections 21,22 of Text 2.)
Unit 4
Factorisation of polynomials over a field. Homomorphisms and factor rings. Prime and maximal ideals. (Chapter 4 Section 23; Chapter 5Sections 26,27 of Text 2.)
Unit 5
Modules. (Chapter 14 Sections 1-5 of Text 1.)

Text Books:
1. P.B. Bhattacharya, S.K. Jain, S.R. Nagpal – Basic Abstract Algebra (2^{nd} edn., 1975)
2. J. B. Fraleigh – A First Course in Abstract Algebra- Narosa (7^{th} edn., 2003)

Reference:
1. I.N. Herstein – Topics in Algebra- Wiley Eastern
2. J.A.Gallian – Contemporary Abstract Algebra
3. Hoffman & Kunze – Linear Algebra – Prentice Hall
4. M. Artin, Algebra, Prentice Hall, 1991

MAT C 102: LINEAR ALGEBRA

Unit I
Linear Transformations: Linear Transformations, The Algebra of Linear Transformations,
Isomorphism, Representation of Transformation by Matrices, Linear Functional, The Double Dual, The Transpose of a Linear Transformation.
(Chapter-3; Sections 3.1, 3.2,3.3, 3.4, 3.5, 3.6, 3.7)
Unit II
Polynomials: Algebras,The Algebra of Polynomials,Polynomial Ideals,The Prime Factorization of a Polynomial.
Determinants:Commutative Rings,Determinant Functions,Permutations and the Uniqueness od Determinants,Additional Properties of Determinants
(Chapter 4 :Sections 4.1,4.2,4.4,4.5,Chapte 5: Sections 5.1,5.2,5.3,5.4)
Unit III

Elementary Canonical Forms: Introductions, Characteristic Values, Annihilating Polynomials Invariant Subspace, Simultaneous Triangulations& Simultaneous Diagonalisation.
(Chapter-6: Section 6.1, 6.2,6.3, 6.4, 6.5, 6.6)
Unit IV
Elementary Canonical Forms: Invariant Direct Sums, The Primary Decomposition Theorem.
The Rational and Jordan Forms: Cyclic Subspaces and Annihilators, Cyclic Decomposition and the Rational Forms, The Jordan Forms.
(Chapter-6: Sections 6.7, 6.8; Chapter-7: Sections: 7.1, 7.2, 7.3 (Omit Proof of the theorems in this (7.3) section)
Unit V
Inner Product Spaces: Inner Products, Inner Product Spaces,Linear Functionals and Adjoints,Unitary Operators,Normal Operators.
Spectral Theory ,Further Properties of Normal Operatorsb
(Chapter-8: Sections 8.1, 8.2,8.3,8.4,8.5 ,Chapter 9 :Sections 9.5,9.6)

Text Book: Kenneth Hoffman & Ray Kunze; Linear Algebra; Second Edition, Prentice-Hall of India Pvt. Ltd

Reference:
1. Stephen H. Friedberg, Arnold J Insel and Lawrence E. Spence: Linear Algebra: 4th Edition 2002: Prentice Hall.
2. Serge A Land: Linear Algebra; Springer
3. Paul R Halmos Finite-Dimensional Vector Space; Springer 1974.
4. McLane & Garrell Birkhoff; Algebra; American Mathematical Society 1999.
5. Thomas W. Hungerford: Algebra; Springer 1980
6. Neal H.McCoy & Thomas R.Berger: Algebra-Groups, Rings & Other Topics: Allyn & Bacon.
7. S Kumaresan; Linear Algebra A Geometric Approach; Prentice-Hall of India 2003.

MAT C 103 Differential Equations I

Unit 1
Existence and Uniqueness of solutions of differential equations. Oscillation theory (Chapter 11 and Chapter 4)
Unit 2
Power series solutions and special functions. Second order linear equations, Gauss's hyper geometric equation, point at infinity (Chapter 5, Section 27-31)
Unit 3
Legendre polynomials, Bessel functions and their properties. Application of Legendre polynomial to potential theory (Chapter 6, Sections 32-35, Appendix A)
Unit 4
Systems of first order equations: linear and nonlinear systems (Chapter 7)
Unit 5

Nonlinear equations: Autonomous systems, phase plan, critical points and stability (Chapter 8, Sections 40-43)

Text Book:

George F. Simmons – Differential Equations with applications and historical notes. Tata McGraw Hill, 1995

References:

1. Birkhoff G &G.C. Rota – Ordinary Differential Equations – Wiley
2. E.A. Coddington – An introduction to Ordinary Differential Equations – Prentice Hall India
3. Chakrabarti – Elements of Ordinary Duifferential Equations & Special Functions – Wiley Eastern

MAT C 104 Real Analysis

Unit 1

Differentiation, Derivative of a real function. Mean value theorems, Continuity of derivatives. L Hospital's rule. Derivatives of higher order. Taylor's theorem. Differentiation of vector valued functions

Unit 2

Reimann – Stieltjes integral. Definition and existence of the integral. Integration and differentiation. Integration of vector – valued functions. Rectifiable curves.

Unit 3

Sequences and series of functions. Uniform convergence. Uniform convergence and continuity. Uniform convergence and differentiation.

Unit 4

Equicontinuous families of functions. Stone – Weierstrass theorem

Unit 5

Some special functions. Power series. The exponential and logarithmic functions. The trigonometric functions. The algebraic completeness of the complex field. Fourier series. The gamma functions.

Text Book:

Walter Rudin – Principles of Mathematical Analysis (3rd edition) – Mc Graw Hill, Chapters 5,6,7 and 8.

References:

1. T.M. Apostol – Mathematical Analysis (2nd edition) – Narosa
2. B.G. Bartle – The Elements of Real Analysis – Wiley International
3. G.F. Simmons – Introduction to Topology and Modern Analysis – McGraw Hill

MAT C 105 Topology

Unit 1
Naïve set theory. Topological spaces. Connected compact spaces.
Unit 2
Continuous functions. Product spaces. The Tychnoff theorem
Unit 3
Separation axioms. Separation by continuous functions. More separability.
Unit 4
Complete metric spaces. Applications.
Unit 5
Nets and filters. Convergence of nets. Convergence of filters. Ultra filters and compactness.
Text (for units 1 to 4):
M. Singer and J.A. Thorpe – Lecture Notes on Elementary Topology and Geometry, Springer Verlag 1967 (Chapter 1 and 2)
(For unit 5):
K.D. Joshi – Introduction to General Topology, Wiley Eastern (1983) (Chapter 10)

References:

1. K.D. Joshi – Introduction to General Topology, Wiley Eastern (1983)
2. G.F. Simmons – Introduction to Topology & Modern Analysis – McGraw Hill
3. J.R. Munkres – Topology, a First Course, Prentice Hall India
4. Kelley J.L. – General Topology, von Nostrand

MAT C 201Complex Analysis

Unit 1
Conformal mapping, linear transformations, cross ratio, symmetry, oriented circles, families of circles, use of level curves, elementary mappings and Riemann surfaces (Chapter 3, Sections 2,3,4)

Unit 2
Complex integration, rectifiable curves, Cauchy's theorem for rectangle and disc, Cauchy's integral formula, higher derivatives (Chapter 4, Sections 1,2)
Unit 3
Local properties of analytic functions, removable singularities, Taylor's theorem, zeroes and poles, local mapping, the maximum principle (Chapter 4, Section 3)
Unit 4

Chains and cycles, simple connectivity, homology, general form of Cauchy's theorem, locally exact differentials, multiply connected regions, residue theorem, argument principle, evaluation of definite integrals (Chapter 4, Sections 4,5)

Unit 5

Harmonic functions, mean value property, Poisson formula, Schwarz theorem, reflection principle, Weierstrass theorem, Taylor series and Laurent series (Chapter 4, Section 6 and Chapter 5, Section 1)

Text Book:

L.V.Ahlfors – Complex Analysis (3rd edition) – McGraw Hill International (1979)

References:

1.Conway J.B. – Functions of One Complex Variable – Narosa
2. E.T.Copson – An Introduction to the Theory of Complex Variables - Oxford

MAT C 202 Functional Analysis

Unit 1

Normed spaces. Continuity of linear maps. Hahn-Banach theorem. Banach spaces

Unit 2

Uniform boundedness principle, closed graph theorem and open mapping theorem, bounded inverse theorem

Unit 3

Duals and transposes. Duals of $L_p([a,b])$ and $C([a,b])$. Weak and weak* convergence

Unit 4

Reflexivity.

Unit 5

Inner product spaces. Orthonormal sets. Approximation and optimization. Projection and Riesz representation theorem.

Text: B.V. Limaye – Functional Analysis (2nd edition) – New Age International, 1996.(Chapters 2,3,4 and 6)

Reference Books

1. E. Kreyszig, Introductory Functional Analysis with Applications (Addison-Wesley)
2. B. Bollabas, Linear Analysis, Cambridge University Press (Indian Edition), 1999.
3. G. F. Simmons, Introduction to Topology and Modern Analysis, McGraw Hill, 1963.
4. A. E. Taylor, D. C. Lay, Introduction to Functional Analysis 2nd edition, Wiley New York, 1980.
5. M.T. Nair, Functional Analysis: A First Course, Wiley Eastern, 1981

MAT C 203 Algebra II

Unit 1
Unique factorization domains, Euclidean domains; Gaussian integers, multiplicative norms. (Chapter 9 Sections 45,46,47 of Text 1.)
Unit 2
Introduction to extension fields. Algebraic extensions.Geometric constructions. (Chapter 6 Sections 29, 31, 32.)

Unit 3
Finite fields. Automorphisms and fields ,The isomorphism extension theorem,. (Chapter 6 Section33, Chapter 10 Sections 48,49 of Text 1.)

Unit 4
splitting fields, separable extensions,Totally inseparable extensions(Chapter 10 Section50,51,52)
Unit 5
Galois theory,Illustrations of Galois theory,Cyclotomic Extensions ,Insolvability of Quintic(Chapter 10 Section53,54,55,56)
Text Books:

1. Fraleigh – A First Course in Abstract Algebra- Narosa (7^{th} edn.), 2003

Reference:

1. J.A.Gallian – Contemporary Abstract Algebra
2. Hoffman & Kunze – Linear Algebra – Prentice Hall
3. P.B. Bhattacharya, S.K. Jain, S.R. Nagpal – Basic Abstract Algebra
4. M. Artin – Algebra, Prentice Hall, 1991

MAT C 204 Differential Equations II

Unit 1
First order partial differential equations (PDE): Curves and surfaces, classification of integrals, linear equations, Pfaffian equations, compatible systems, Charpit's Method, Jacobi's method. (Chapter 1 Sections 1.7-1.8)
Unit 2
Integral surfaces through a given curve, quasilinear equations, nonlinear equations (Chapter 1 Sections 1.9,1.10,1.11)
Unit 3
Second order equations: classification, one-dimensional wave equation (Chapter 2 Sections 2.1,2.2,2.3)

Unit 4
Laplace's equation (Chapter 2 Section 2.4)
Unit 5

Heat conduction problem, Duhamel's principle, families of equipotential surfaces, Kelvin's inversion theorem (Chapter 2 Sections 2.5,2.6,2.8,2.9)

Text :

T. Amarnath – An elementary course in partial differential equations (2nd edition) – Narosa Publishing House, 2003

References:

1. Ian Sneddon – Elements of partial differential equations, McGraw Hill, 1983
2. Phoolan Prasad and Renuka Ravindran – Partial differential equations, New Age

MAT C 205 Measure and Integration

Unit 1
Introduction. Measurable functions. Measures.
Unit 2
The integral. Integrable functions. L_p – spaces.
Unit 3
Modes of convergence.
Unit 4
Generation of measures. Decomposition of measures.

Unit 5
Product measures.

Text:

R.G. Bartle – The Elements of Integration (1966), John Wiley & Sons (Complete Book)

References:

1. H.L. Royden – Real Analysis –Macmillan
2. de Barra – Measure and Integration
3. Inder K. Rana – Measure and Integration – Narosa

MAT C 301 Differential Geometry

1. Level sets, vector fields, tangent spaces, surfaces, orientation, Gauss map.
2. Geodesics, parallel transport, Weingarten map, curvature of plane curves.
3. Arc length, line integrals, curvature of surfaces.
4. Parametrized surfaces. Local equivalence of surfaces and parametrized surfaces.
5. Rigid motions and congruence, isometries.

Text Book:

T. A. Thorpe – Elementary Topics in Differential Geometry, Springer-Verlag, Chs.1-12, 14,15,22 and 23.

References:

1. Guillemine & Pollack – Differential Geometry, Prentice Hall
2. Struik D.J. – Classical Differential Geometry – Dover (2nd edn.) (1988)
3. Kreyszig, E. – Introduction to Differential Geometry and Riemannian Geometry – Univ. of Toronto Press (1969)
4. M. Spivak – A Comprehensive Introduction to Differential Geometry Vols. 1-3, Publish or Perish Boston (3rd edn.) (1999)

MAT C 302 Fuzzy Mathematics

1. From classical (crisp) sets to fuzzy sets: characteristics and significance of the paradigm shift. Additional properties of α-cuts. Representation of fuzzy sets. Extension principle for fuzzy sets. (Chs. 1 & 2 of the Text Book)
2. Operations on fuzzy sets. Types of operations. Fuzzy complements. t-norms, t-conorms. Combinations of operations. Aggregate operations. (Ch. 3 of the Text)
3. Fuzzy arithmetic. Arithmetic operations on intervals. Arithmetic operations on fuzzy numbers. Lattice of fuzzy numbers. Fuzzy equations. Fuzzy decision making (Chs. 4 & 15 of the Text)
4. Fuzzy relations. Fuzzy equivalence, compatibility and ordering relations. Fuzzy morphisms. sup-i, inf- $\omega_i$compositions of fuzzy relations. (Ch. 5 of the Text)
5. Fuzzy logic. Fuzzy propositions. Fuzzy quantifiers. Linguistic hedges. Inference from conditional, conditional and qualified and quantified propositions. (Ch. 8 of the Text)

Text Book:

Fuzzy sets and Fuzzy logic Theory and Applications – G. J. Klir & Bo Yuan – PHI (1995) Chs. 1,2,3,4,5,8 and 15

References :

1. Zimmermann H. J. – Fuzzy Set Theory and its Applications, Kluwer (1985)
2. Zimmermann H. J. – Fuzzy Sets, Decision Making and Expert Systems, Kluwer (1987)
3. Dubois D. & H. Prade – Fuzzy Sets and Systems: Theory and Applications – Academic Press (1980)

MAT C 303 Probability

Unit 1

Probability spaces – Dynkin's theorem, construction of probability spaces, measure constructions. (sections 2.1 to 2.6)

Unit 2

Random variables, elements, and measurable maps – inverse maps, measurable maps, induced probability measures, measurability and continuity, measurability and limits, fields generated by maps.(Sections 3.1, 3.2 except 3.2.2, 3.3 and 3.4) Independence – records, ranks, Renyi theorem, groupings, zero-one laws, Borel-Contelli lemma (Sections 4.1 to 4.6)

Unit 3

Integration and expectation – limits and integrals, infinite integrals the transportation theorem and densities, product spaces, independence and Fubini theorem, probability measures on product spaces. (Sections 5.1 to 5.10 except 5.6)

Unit 4

Convergence concepts – almost sure, convergence in probability, quantile estimation, Lp convergence(sections 6.1 to 6.6 except 6.2.1 and 6.4)

Unit 5

laws of large numbers. General weak law of large numbers, almost sure convergence of sums of independent random variables (Sections 7.1 to 7.3) Strong law of large numbers, applications of SLLN, Kolmogorov three series theorem (statement only)

Text:

Sidney I Resnick – A Probability Path, Birkhauser (1999) (Chapters 2 to 7)

References:

1. K.L. Chung – Elementary Probability Theory, Narosa
2. W. Feller – Introduction to Probability Theory and Applications volumes & II, John Wiley, 1968
3. A. K. Basu, Measure and Probability, PHI (2004)

ELECTIVES

1. ALGEBRAIC GEOMETRY

1. Affine algebraic varieties. The Zariski topology. Morphisms. Dimension. Hilbert basis theorem. Hilbert Nullstellensatz. The co-ordinate ring. The spectrum of a ring
2. Projective space. Projective varieties. Projective closure. Morphisms of projective varieties. Automorphisms of projective space. Quasi-projective varieties. A basis for the Zariski topology. Regular functions.
3. Classical constructions.
4. Smoothness. Bertini's theorem. The Gauss mapping.
5. Birational geometry. The classification problems.

Text book:

(An invitation to) Algebraic Geometry – K. E. Smith, L. Kahnpaa, P.Kekalainen and W. Treves, Springer (2000) Chapters 1 to 7.

References:

1. Undergraduate Commutative Algebra – Miles Reid, Cambridge Univ. Press. (1995)

2. Introduction to Commutaive Algebra – M. F. Atiyah & I. G. MacDonald – Addison-Wesley (1969)

3. Algebraic Geometry – Keith Kendig – Springer

4. Undergraduate Algebraic Geometry – Miles Reid, Cambridge Univ. Press (1988)

5. Hartshorne, R. – Algebraic Geometry, Springer-Verlag (1977)

6.Shafarevich I. R. – Basic Algebraic Geometry, Springer-Verlag (1974).

2 PROJECTIVE GEOMETRY

1. What is Projective Geometry? Projectivities. Perspectivities. Triangles and quadrangles: axioms and simple consequences. Perspective triangles. Quadrangular sets. Harmonic sets. The principle of duality. The Desargues' configuration. The invariance of the harmonic relation. .Trilinear polarity. Harmonic nets..

2. The fundamental theorem and Pappus' theorem: The axis of a projectivity. Pappus and Desargues. One-dimensional projectivities. Superposed ranges. Parabolic projectivities. Involutions and hyperbolic involutions. two dimensional projectivities. Projective, perspective and involutory collineations. Projective correlations.

3. Polariies: Conjugate points and conjugate lines. polar triangles. The use of self-polar pentagon. A self-conjugate quadrilateral. Product of two polarities. Self-polarity of the Desargues configuration. The conic. The polarity induced by a conic. Projectivity related pencils. Steiner's definition for a conic. The conic touching five given lines. The conic through five given points. Conics through four given points. Degenerate conics.

4. A finite projective plane. S combinatorial scheme for PG(2,5). Involution. Collineation and correlation conic. Is the circle a conic? Affine space. the language of pencils. The plane at inifinity. Euclidean space.

5. Co-ordinates. The idea of analytic geometry. Projective collineations. Polarities.. Conics. The analytic geometry of PG(2,5). Cartesian coordinates. Planes of characteristic two.

Text Book:

H.S.M. Coxeter – Projective Geometry (2nd edn.)—Univ. of Toronto Press (1974) . (The whole book).

References:

1. Struik D. J.– Lectures on Analytic and Projective Geometry – Addison-Wesley, 1953
2. Coxeter H. S. M.- The real Projective Plane (1955)

3 ADVANCED COMPLEX ANALYSIS

1. Partial fractions. Infinite products. Canonical products. The Gamma function. Stirling's formula. Entire functions. Jensen's fomula. Hadamard's theorem (without proof)

2. Riemann mapping heorem. Boundary behaviour. Use of reflection principle. Analytic arcs. Conformal mapping of polygons. The Schwarz-Christoffel formula. Mapping on a rectangle. The triangle functions of Schwarz. (Ch. 6 Sections 1,2)

3. Functions with men value property. Harnack's principle. Subharmonic functions. Solutions. Solution of Dirichlet problem (Ch. 6 Sections 3,4)

4. Simply periodic functions. Doubly periodic functions. Unimodular transformations. Canonical basis. Generl properties of elliptic functions. The Weierstrass ρ-function. The functions $\zeta(z)$ and $\sigma(z)$. The modular function $\lambda(\tau)$. Conformal mapping by $\lambda(\tau)$. (Ch. 7)

5. Analytic continuation. Germs and sheaves. Sections and Riemann surfaces. Analytic continuations along arcs. Monodromy theorem. Branch points. Picard's theorem. (Ch. 8 Sections 1,3)

Text Book:

Ahlfors L. V. – Complex Analysis (3rd edition) McGraw Hill International.

References:

1. Conway J. B. – Functions of one complex variable – Narosa (2002)
2. Lang S.– Complex Analysis – Springer (3rd edn.) (1995)
3. Karunakaran, V. – Complex Analysis – Alpha Science International Ltd. (2nd edn.) 2005.

4 ANALYTICAL MECHANICS

UNIT – I Equation of Motion, Conservation Laws

UNIT – II Integration of the equations of motion. Free Oscillation in one dimension

UNIT – III Motion of a rigid body

UNIT – IV Hamilton's equations, Routhian, Poisson brackots, Action as a function of the coordinates, Maupertius principles

UNIT – V Canonical transformations, Liouville's theorem, Hamilton – Jacobi equation, Separation of variables. Adiabatic invariant canonical variables

Text book:

L.D. Landau and E.M. Lifshitz-Mechanics (3rd Edition) Porgamon 1976 [Chapters 1, 2, 3, 5, 21, 6, 7.40 – 7.50]

References:

1. Herbert Goldstein (1980), Classical Mechanics, 2nd Ed., Narosa)
2. N.C. Rana and P.S. Joag (1991), Classical Mechanics, Tata Mc Graw Hill
3. K.C. Gupta (1988), Classical Mechanics of Particles and Rigid Bodies, Wiley Eastern
4. F.R. Gantmacher (1975), Analytical Mechanics, MIR Publishers.

5 STOCHASTIC PROCESSES

Unit 1: A brief description of Markov Process, Renewal Process, Stationary Process. Markov Chains: n-step transition probability matrix, classification of states, canonical representation of transition probability matrix, finite Markov chains with transient states. Irreducible Markov Chains with ergodic states: Transient and limiting behaviour.

Unit 2: First passage and related results. Branching Processes and Markov chains of order larger than 1, Lumpable Markov Chains, Reversed Markov Chains.

Unit 3: Applied Markov Chains: Queuing Models, Inventory Systems, Storage models, Industrial Mobility of Labour,, Educational Advancement, Human Resource Management, Term Structure, Income determination under uncertainty, Markov decision process. Markov Processes: Poisson and Pure birth processes, Pure death processes, Birth and death processes, Limiting distributions.

Unit 4: Markovian Networks. Applied Markov Processes: Queueing models, Machine interference problem, Queueing networks, Flexible manufacturing systems, Inventory systems, Reliability models, Markovian Combat models, Stochastic models for social networks; Recovery, relapse and death due to disease.

Unit 5: Renewal Processes: Renewal processes in discrete and continuous time, Alternating renewal process, Markov renewal and semi-Markov processes, Renewal reward processes.

Text book:

U.N. Bhat and Gregory Miller: Elements of Applied Stochastic Processes, Wiley Interscience, 2002 (Chs. 1, 2, 3, 4, 6, 7, 9.1-9.4, 9.9 and 10)

References:

1. Karlin and Taylor: A First Course in Stochastic Processes, Academic Press, 1975
2. E.Parzen: Stochastic Processes, Wiley 1968.
3. J.Medhi: Introduction to Stochastic Processes, New Age International Publishers, 1994, Reprint 1999.

6 FLUID MECHANICS

1. Introduction. The continuum hypothesis, volume forces and surface forces, transportation phenomenal, real fluids and ideal fluids. Poperties of gases and liquids. Pressure and thrust. Kinematics of the flow. Differentiation following the motion of the fluid. Velocity of a fluid at a point. Lagrangian and Eulerian description. Acceleration. Equation of continuity, stream lines and path lines.

2. Equation of motion of an ideal fluid. Euler's equation of motion. Bernoulli's equation, surface condition, velocity potential. Irrotational and rotational motion.
3. Particular method and applications. Motion in two dimensions, stream function, complex potential. Sources and sinks, Doublets. Images, conformal transformation and its application in Fluid Mechanics. Elements of vortex motion. vorticity and related theorems, line vortices, vortex street. Karman vortex street. Helmholtz theorems on vorticity.

4. General theory of irrotational motion. Flow and circulation, constancy of circulation. Minimum kinetic energy. Motion of cylinders. Forces on a cylinder, Theorem of Blasius, Theorems of Kutta and Joukowski. Axi-symmetric flows. Stokes stream function, motion of sphere.

5. Viscous flow theory. Expression for the stress tensor, stress-strain relation for a Newtonian fluid, the coefficient of viscosity. Navier-Stokes equation of motion, Energy dissipation, Equation satisfied by stream function in 2-D motion. Flow between parallel planes. Flow in a pipe, Flow between rotating cylinders.

Text Books:

1.Frank Chorlton – A Text Book of Fluid Dynamics – ELBS and Van Nostrand (1967) Chs. 2-5 and Ch.8

2. R. von Mises and K. O. Friedricks – Fluid Dynamics – Springer Verlag (1971)

Reference Books:

1)Davies – Modern Developments in Fluid Mechanics vol I & II – Van Nostrand

2) W.H.Besant and A.R.Ramsey – A Treatise on Hydromechanics Part II – ELBS

3) L.M.Milne Thomson – Theoretical Hydrodynamics Mac Millan (1962)

7 ALGEBRAIC TOPOLOGY

1. Geometric complexes and polyhedra. Orientation of geometric complexes.

2. Simplicial homology groups. Structure of homology groups. The Euler-Poincare theorem. Pseudomanifolds and the homology groups of Sn.

3. Simplicial approximation. Induced homomorphisms on homology groups. Brouwer fixed point theorem and related results.

4. The fundamental groups. Examples. The relation between H1(K) and π1(|K|).

5. Covering spaces. Classification. Universal covering spaces. Applications.

Text Book:

Fred H. Croom – Basic Concepts of Algebraic Topology – Springer Verlag (1978)

References:

1) Maunder – Algebraic Topology – Van Nostrand-Reinhold (1970)
2) Munkres J.R. – Topology, A First Course – Prentice Hall (1975)

8 NUMERICAL ANALYSIS AND COMPUTING

1 Principles of Numerical Calculations

1.1 Common Ideas and Concepts, Fixed-Point Iteration, Newton's Method, Linearization and Extrapolation, Finite Difference Approximations,

1.2 Some Numerical Algorithms, Solving a Quadratic Equation, Recurrence Relations. Divide and Conquer Strategy.

1.3 Matrix Computations, Matrix Multiplication, Solving Linear Systems by LU Factorization, Sparse Matrices and Iterative Methods, Software for Matrix Computations.

1.4 The Linear Least Squares Problem, Basic Concepts in Probability and Statistics, Characterization of Least Squares Solutions, The Singular Value Decomposition, The Numerical Rank of a Matrix

1.5 Numerical Solution of Differential Equations, Euler's Method , Introductory Example, Second Order Accurate Methods.

2. How to Obtain and Estimate Accuracy

2.1 Basic Concepts in Error Estimation, Sources of Error, Absolute and Relative Errors, Rounding and Chopping.

2.2 Computer Number Systems, The Position System, Fixed- and Floating-Point Representation, IEEE Floating-Point Standard.,Elementary Functions, Multiple Precision

2.3 Accuracy and Rounding Errors, Floating-Point Arithmetic, Basic Rounding Error Results, Statistical Models for Rounding Errors, Avoiding Overflow and Cancellation.

3. Interpolation and Approximation

3.1 The Interpolation Problem, Bases for Polynomial Interpolation, Conditioning of Polynomial

3.2 Interpolation Formulas and Algorithms, Newton's Interpolation ,Inverse Interpolation, Barycentric Lagrange Interpolation, Iterative Linear Interpolation, Fast Algorithms for Vandermonde Systems, The Runge Phenomenon

3.3 Generalizations and Applications, Hermite Interpolation, Complex Analysis in Polynomial Interpolation, Rational Interpolation, Multidimensional Interpolation.

3.4 Piecewise Polynomial Interpolation, Bernštein Polynomials and Bézier Curves, Spline Functions, The B-Spline Basis, Least Squares Splines Approximation. The Fast Fourier Transform. The FFT Algorithm.

4. Numerical Integration

4.1 Interpolatory Quadrature Rules ,Treating Singularities, Classical Formulas, Super-convergence of the Trapezoidal Rule, Higher-Order Newton–Cotes' Formulas

4.2 Integration by Extrapolation , The Euler–Maclaurin Formula, Romberg's Method, Oscillating Integrands Adaptive Quadrature

5. Solving Scalar Nonlinear Equations

5.1 Basic Concepts and Methods, the Bisection Method, Limiting Accuracy and Termination, Fixed-Point Iteration, Convergence Order and Efficiency

5.2 Methods Based on Interpolation, Method of False Position, The Secant Method, Higher-Order Interpolation Methods.

5.3 Methods Using Derivatives, Newton's Method, Newton's Method for Complex Roots, An Interval Newton Method, Higher-Order Methods

5.4 Finding a Minimum of a Function
Unimodal Functions and Golden Section Search, Minimization by Interpolation.

5.5 Algebraic Equations, Elementary Results, Ill-Conditioned Algebraic Equations, Three Classical Methods, Deflation and Simultaneous Determination of Roots, Modified Newton Method.Sturm Sequences, Finding Greatest Common Divisors

Note : Practicals should be devised on the topics mentioned above. The relevant algorithms can be taken from Text Book 2, and implemented on a computer. A minimum of 20 algorithms should be implemented during the semester. The following topics can also be included in the practicals.

A.1 Vectors and Matrices A-1
Linear Vector Spaces , Matrix and Vector Algebra , Rank and Linear Systems ,Special Matrices
A.2 Sub matrices and Block Matrices ,Block Gaussian Elimination A.3 Permutations and Determinants
A.4 Eigenvalues and Norms of Matrices,The Characteristic Equation , The Schur and Jordan Normal Forms , Norms of Vectors and Matrices

Note: The Instructor should provide good problems and computer exercises to bring out the concepts clearly and to develop computational skills efficiently. MATLAB/C language may be used as a platform for developing skills. The Numerical recipe by Text Book 2, which is freely downloadable, may also be used for developing programming skills. But use it for true learning purpose.

Text Books:

1. Numerical Analysis in Scientific Computing (Vol.1) Germund Dahlquist, Cambridge University press)
2. Numerical Recipes in C – The art of scientific computing (3rd edn.)(2007) – William Press (also available on internet)

Reference Books:

1. Applied Numerical Analysis using MATLAB. (2nd Edn) Laurene Fausett (Pearson)
2. Numerical Analysis: Mathematics of Scientific Computing, David Kincaid, Chency et.al. , Cengage Learning (Pub), 3rd Edn
3. Deuflhard P. & A. Hofmann – Numerical Analysis in Modern Scientific Computing – Springer (2002).

9 GRAPH THEORY

1. Basic results. Directed graphs. (Chapters I and II of the Text)
2. Connectivity. Trees (Chs. III and IV)
3. Independent sets and matchings. Eulerian and Hamiltonian graphs. (Chs. V and VI)
4. Graph colourings (Ch. VII)
5. Planarity (Ch. VIII)

Text Book:

R. Balakrishnan, K. Ranganathan – A Text Book of Graph Theory – Springer (2000)

References:

1.C. Berge – Graphs and Hypergraphs – North Holland (1973)

2.J. A. Bondy and V. S. R. Murty – Graph Theory with Applications, Mac Millan 1976

3.F. Harary – Graph Theory – Addison Wesley, Reading Mass. (1969)

4.K. R. Parthasarathy – Basic Graph Theory – Tata McGraw Hill (1994)

10 FRACTAL GEOMETRY

1. Mathematical background. Hausdorff measure and dimension (Chapters 1 and 2 except sections 2.4 and 2.5)
2. Alternate definitions of dimension (Ch. 3)
3. Local structure of fractals. Projections of fractals (Chs. 5 and 6)
4. Products of fractals. Intersection of fractals (Chs. 7 and 8)
5. Fractals defined by transformations, self-similar sets and self-affine sets (Ch.9)

Text Book:

Frctal Geometry, Mathematical Foundations and Applications by Kenneth Falconer, John Wiley (1990)

Reference:

Mandelbrot B. B. – The Fractal Geometry of Nature – Freeman (1982).

11 CODING THEORY

1. Introduction to Coding Theory. Correcting and detecting error patterns. Weight and distance. MLD and its reliability. Error-detecting codes. Error correcting codes. (Chapter 1 of the Text)
2. Liner codes. Generating matrices and encoding. Parity check matrices. Equivalent codes. MLD for linear codes. reliability of IMLD for linear codes (Ch. 2)
3. Perfect codes. Hamming codes. extension codes. Decoding of extended Golay code. Reed-Muller codes. Fast decoding of RM(1,m) (Ch. 3)
4. Cyclic linear codes. Generating and parity check matrices for cyclic codes. Finding cyclic codes. Dual cyclic codes. (Ch. 4)
5. BCH codes. Decoding 2-error-correcting BCH code. Reed-Solomon codes. Decoding.(Ch. 5 and sections 6.1, 6.2 and 6.3)

Text Book:

Coding Theory and Cryptography The Essentials (2nd edition) – D. R. Hankerson, D. G. Hoffman, D. A. Leonard, C. C. Lindner, K. T. Phelps, C. A. Rodger and J. R. Wall – Marcel Dekker (2000)

Reference Books:

1.J. H. van Lint – Introduction to Coding Theory – Springer Verlag (1982)

2.E. R. Berlekamp – Algebraic Coding Theory – McGraw Hill (1968)

12 CRYPTOGRAPHY

1. Classical cryptography. Some simple cryptosystems. Cryptanalysis (Chapter 1 of the Text)
2. Shannon's theory (Ch. 2)
3. Block ciphers and the advanced encryption standard (Ch. 3)
4. Cryptographic hash function. (Ch. 4)
5. The RSA cryptosystems and factoring integers (Sections 5.1-5.6 of Ch.5)

Text book:

Cryptography, Theory and Practice – Douglas R. Stinson – Chapman & Hall (2002)

References:

1. N. Koblitz – A Course in Number Theory and Cryptography (2nd edition) Springer Verlag (1994)
2. D.R.Hankerson etc. – Coding Theory and Cryptography The Essentials – Marcel Dekker

13 NUMBER THEORY

1. Basic representation theorem. The fundamental theorem of arithmetic; combinatorial and computational number theory: Permutations and combinations, Fermat's little theorem, Wilson's theorem, Generating functions; Fundamentals of congruences- Residue systems, Riffling; Solving congruences- Linear congruences, Chinese remainder theorem, Polynomial congruences.

2. Arithmetic functions- combinatorial study of phi (n), Formulae for d (n) and sigma (n), multivariate arithmetic functions, Mobius inversion formula; Primitive roots- Properties of reduced residue systems, Primitive roots modulo p; Prime numbers- Elementary properties of Pi (x), Tchebychev's theorem.

3. Quadratic congruences: Quadratic residues- Euler's criterion, Legendre symbol, Quadratic reciprocity law; Distribution of Quadratic residues- consecutive residues and nonresidues, Consecutive ttriples of quardratic residues.

4. Additivity: Sums of squares- sums of two squares, Sums of four squares; Elementary partition theory- Graphical representation, Euler's partition theorem, Searching for partition identities; Partition generating functions- Infinite products as generating functions, Identities between infinite series and products.

5. Partition Identities- Euler's pentagonal number theorem, Rogers-Ramanujan identities, Series and products identities, Schur's theorem; Geometric Number Theory: Lattice points- Gauss's circle problem, Dirichlet's divisor problem.

Text Book

George E Andrews: Number Theory, Dover Publications (1971) Chs.3 to 15.

References

1. Andre Weil-Basic Number Theory (3rd edn.) Springer-Verlag (1974)
2. Grosswald, E.-Introduction to Number Theory Brikhauser (2nd edition) 1984.

14. ANALYTIC NUMBER THEORY

1. The Fundamental theorem of Arithmetic, Arithmetical functions and Dirichlet multiplications
2. Averages of arithmetical functions. Some elementary theorems on the distribution of prime numbers.
3. Congruences. Finite abelian groups and their characters.
4. Dirichlet's theorem on primes in arithmetic progressions. Periodic Arithmetical Functions and gauss sums
5. Quadratic residues and quadratic reciprocity law. Primitive roots.

Text Book

Tom M. Apostol- Introduction to Analytic Number Theory (Springer International Edn. 1998) Chapters 1-10

References

1. G.H.Hardy & Wright Introduction to Theory of Numbers (Oxford) 1985
2. H.Davenport- The Higher Arithmetic (Cambridge) (6th edn.) 1992.

15 ALGEBRAIC NUMBER THEORY

1. Algebraic background, Symmetric Polynomials, modules, Free abelian groups, Algebraic numbers, Conjugates and discriminants, algebraic integers, integral basis, norms and traces, Rings of integers.(Sections 1.4-1.6, 2.1-2.6 of the text book)
2. Quadratic fields. Cyclotomic fields. Factorization into irreducible: Historical back ground. Trivial factorization int irreducible (Sections 3.1, 3.2, 4.1-4.3 of the text book)
3. Examples of non-unique factorization into irreducible. Prime factorization, Euclidean domains, Euclidean quadratic fields. Congruences of unique factorization Ramanajuan-Nagell theorem. (Sections 4.4-4.9 of the text book)
4. Ideals, Historical background, Prime factorization of ideals. The norm of an ideal. Non-unique factorization in cyclotomic fields. Lattices. The quotient torus. (Sections 5.1-5.4, 6.1, 6.2 of the text book)
5. Minkoswki's theorem. The two-square theorem, The four-square theorem. Geometric representation of algebraic numbers. The space Lst, (Sections 7.1-7.3,6.1 of the text book)

Text Book

1.N.Stewart & D.O.Tall-Algebraic Number Theory (2nd edn.) Chapman & Hall (1987)

References:

1. P.Samuel- Theory of Algebraic numbers-Herman Paris Houghton Mifflin (1975)
2. S Lang-Algebraic Number Theory-Addison Wesley (1970)

16 ADVANCED FUNCTIONAL ANALYSIS

Unit I:
Spectrum of a bounded linear operator. Compact linear maps. Spectrum of a compact linear operator. (Chapter 3 Section 12, Chapter 5 Sections17,18.)
Unit II:
Fredholm alternatives. Approximate solutions. (Chapter 5 Sections19,20.)
Unit III:
Bounded operators and adjoints. Normal, unitary, self-adjoint operators. Spectrum and numerical range. (Chapter7 Sections 25-27)
Unit IV:
Compact self-adjoint operators. (Chapter 7 Section 28.)
Unit V:
Sturm-Liouville problems. (Appendix C)
Text Book:
B.V. Limaye, Functional Analysis, Second Edition, New Age International, 1996.

Reference Books:

1. B. Bollabas, Linear Analysis, Cambridge University Press (Indian edition), 1999.
2. E. Kreyzig, Introductory Functional Analysis with Applications, John Wiley and Sons, 2001.
3. A.E. Taylor and D.C. Lay, Introduction to Functional Analysis, 2nd ed., Wiley, New York, 1980.
4. M. T. Nair, Functional Analysis: A First Course, Prentice-Hall of India, 2002 (Reprinted: 2008)

17 OPERATIONS RESEARCH

1. Linear programming in two-dimensional spaces. General LP problem. Feasible, basic and optimal solutions, simplex method, simplex tableau,

finding the first basic feasible solution, degeneracy, simplex multipliers. (Chapter 3 Sections 1-15).

2. The revised simplex method. Duality in LP problems, Duality theorems, Applications of duality, Dual simplex method, summary of simplex methods, Applications of LP. (Chapter 3. Sections 16-22)
3. Transportation and Assignment problems (Chapter 4)
4. Integer programming. Theory of games (Chapters 6 and 12)
5. Kuhn-Tucker theory and non-linear programming. Dynamic programming. (Chapters 8 and 10)

Text Book:

K. V. Mital and C. Mohan – Optimisation Methods in Operations Research and Systems Analysis (3rd edition) _--New Age International (1996).

Reference Books:

1. Wagner – Operations Research, Prentice Hall India
2. A. Ravindran, Don T. Philips, James Solberg – Operations Research, Principles and Practice – John Wiley (3rd edition)
3. G. Hadley – Linear Programming – Addison Wesley
4. Kanti Swarup, P.K.gupta, Man Mohan – Operations Research – S. Chand & Co.

Name: ……………..

Reg. No. : ……………

First Semester MSc.(Mathematics) Model Question Paper

(CCSS-2015 Admission)

Kannur University

MAT C 101: ALGEBRA I

Time: 3 Hours. **Max. Marks**: 60

(Each question carries 12 marks .Answer any **ONE** question from each unit)

UNIT I

1. a) Define simple group and show that M is a maximal normal subgroup of G if and only if G/M is simple. (5)
   b) Show that the group $\mathbb{Z}_m \times \mathbb{Z}_n$ is cyclic and is isomorphic to $\mathbb{Z}_{mn}$ if and only if m and n are relatively prime. (5)
   c) Find ker $\emptyset$ for $\emptyset: Z \times Z \to Z$ where $\emptyset(1,0) = 3$ and $\emptyset(0,1) = -5$. (2)
2. a) Let G be a group and C be the commutator subgroup of G.Then prove that C is a normal subgroup of G.Also prove that , if N is a normal subgroup of G then G/N is abelian iff C≤N. (5)
   b) Let H be a subgroup of a group G.Then prove that left coset multiplication is well defined by the equation (aH)(bH)=(ab)H iff H is a normal subgroup of G. (4)
   c) Prove that A_4 cannot contain a subgroup of order 6. (3)

UNIT II

3. a) State and prove Cauchy's theorem. (5)
   b) Prove that for a prime number p, every group G of order p^2 is abelian (4)
   c) Compute the fixed sets X_σ for each $\in D_4$. (3)
4. a) Derive Burnside's formula. (4)
   b) Prove that the center of a finite nontrivial p-group G is nontrivial. (3)
   c) State and prove first Sylow theorem. (5)

UNIT III

5. a) Show that any integral domain D can be enlarged to a field F. (7)
   b) Let G be a finitely generated abelian group with generating set $\{a_1, a_2, \dots, a_n\}$ .Let $\emptyset: Z \times Z \times \dots \times Z \to G$ be defined by $\emptyset(h_1, h_2, \dots, h_n) = h_1a_1 + h_2a_2 + \cdots + h_na_n$.Then show that $\emptyset$ is a homomorphism onto G. (3)
   c) Define free group generated by the set A (2)
6. a) State and prove the Evaluation homomorphisms for field theory (5)
   b) Let F be a field of quotients of D and let L be any field containing D.Then show that there exist a map $\psi: F \to L$ that gives an isomorphism of F with a subfield of L such (5)

that $\psi(a) = a$ for $\in D$.

(c) Define the rank of a group G. Find the rank of the group $\mathbb{Z} \times \mathbb{Z}$. (2)

UNIT IV

7. a) State and prove division algorithm in F[x.] (5)

b) Let F be a field and f(x),g(x)∈ F[x] . Show that f(x) divides g(x) iff g(x)∈⟨f(x)⟩. (2)

c) Demonstrate that $x^3 + 3x^2 - 8$ is irreducible over Q. (3)

d) Show that if R is a ring with unity and N is an ideal of R containing a unit then N=R. (2)

8. a) State and prove Eisenstein criterion theorem. (4)

b) Check irreducibility of $8x^3 + 6x^2 - 9x + 24$. (2)

c) If F is a field then show that every ideal in F[x] is principal. (3)

d) State and prove factor theorem. (3)

UNIT V

9. a) Define an R-module. Show that the polynomial ring R[x] over a ring R is an R-module (4)

b) If an R-module M is generated by a set $\{x_1, x_2, \ldots, x_n\}$ and 1∈ R ,then prove that $M = \{r_1x_1 + r_2x_2 + \cdots + r_nx_n / r_i \in R\}$ where ,the right side is symbolically written $\sum_{i=1}^{n} Rx_i$. (4)

c) Show that the submodules of the quotient module M/N are of the form U/N where U is a submodule of M 30ontaining N (4)

10. a) Let R be a ring with unity .Prove that an R-module M is cyclic iff M isomorphic to $R/_I$ for some left ideal $I$of R. (4)

b) Define free module and give an example. Let M be a free R-module with a basis $\{ e_1, e_2, \ldots, e_n \}$. Then show that $M \cong \mathbb{R}^n$. (4)

c) state and prove Schur's lemma. (4)

First Semester M.Sc(Mathematics) Model Question Paper
(CCSS 2015 Admission)
Kannur University
MAT C 102 **Linear Algebra**

(Each question carries 12 marks .Answer any **ONE** question from each unit)

Time: 3Hours. Max.Marks:60

Unit – I

1. a) Let V and W be vector spaces over the field F and let T be a linear transformation from V into
   W. Prove that if V is finite dimensional, then rank(T) + nullity T = dim V
   b) Find a basis for the space L(R^2, R^3) over R.

2. a) Let B and B' be two ordered bases for an n dimensional vector space V over the field F and T
   be a linear operator on V. Then prove that there exist an invertible nxn matrix P over F such That $[T]_{B'} = P^{-1}[T]_B P$

   b) Find range an null space of the linear operator T(a,b,c)=(a+b,2c,0) on R^3.

Unit- II

3.a) Let f and d be non-zero polynomials over a filed such that deg d $\leq$ deg f. Show that there exists a
polynomial g in F[x] such that either f-dg=0 or deg(f-dg)<degf

b)Let f be a polynomial with derivative f'.Show that f is a product of distinct irreducible polynomials over F if and only if f and f' are relatively prime.

4.a)Define determinant function.
Let D be an n-linear function on n×n matrices over K.Suppose D has the property that D(A)=0 Whenever two adjacent rows of A are equal.Show that D is alternating.

b)Let K be a commutative ring with identity and let A and B be n×n matrices over K.Show that det(AB)=det(A)det(B).

Unit III

5. a) Prove the Cayley-Hamilton theorem.

b) Let A and B be nxn matrices over a field F. Prove that if I-AB is invertible then I-BA is also invertible. Deduce that AB and BA have the same characteristic values.

6. a) State and prove a necessary and sufficient condition for a linear operator on a finite dimensional vector space to be triangular.

b) Find an invertible matrix P such that $P^{-1} AP$ is a diagonal matrix, where

$$A = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}.$$

Unit IV

7. a) State and prove Primary decomposition theorem.

b) if $T(x_1,x_2)= (2x_1, x_2, -x_1)$ find a diagonalizable operator D and a nilpotent operator N on R^2 such that T=D+N

8. a) Define a cyclic vector for a linear operator T on a vector space. If a linear operator T on a finite dimensional vector space has a cyclic vector, show that dim V is the same as the degree of the minimal polynomial of T.

b) Let T be a diagonalizable operator on an n-dimensional vector space. If T has a cyclic vector show that T has r distinct characteristic values.

Unit V

9. a) Define an inner product space. Prove that an orthogonal set of non-zero vector in an inner product space is linearly independent.

b) If W is a finite dimensional subspace of an inner product space V then show that $V=W\oplus W^{\perp}$ where $W^{\perp}$ is the orthogonal complement of W in V.

10.a)For any linear operator T on a finite dimensional innerproduct space V ,show that there exist a unique linear operator T^* on V such that $(T\alpha \mid \beta)=(\alpha \mid T^*\beta)$ for all α, β in V.

b)Let V be a finite dimensional innerproduct space and T a non-negative operator on V.Then show that T has a unique non-neagative square root ,that is ,there is one and only one non- negative operator N on V such that $N^2 = T$.

Name : ……………..
Reg. No. : ……………

First Semester MSc.(Mathematics) Model Question Paper
(CCSS-2015 Admission)
Kannur University
MAT C 103 : DIFFERENTIAL EQUATIONS I

Time: 3 Hours. **Max. Marks** : 60

(Each question carries 12 marks .Answer any **ONE** question from each unit)

UNIT I

1) State and prove Picard's existence and uniqueness theorem for first order differential equations

2) a) Define Lipschitz condition

b) Show that f(x, y) $=y^{\frac{1}{2}}$

i) does not satisfy a Lipschitz condition on the rectangle $|x| \leq 1$ and $0 \leq y \leq 1$

ii) satisfy Lipschitz condition on the rectangle $|x| \leq 1$ and $c \leq y \leq d$

UNIT II

3) a) Determine the nature of the point x = 0 for

(i) $y'' + (\sin x)y = 0$ (ii) $x^4 y'' + (\sin x) y = 0$

b) Verify that the origin is a regular singular point and calculate two independent Frobenius series solutions of $2xy'' + (x+1)y' + 3y = 0$

4) a) Express $sin^{-1}x$ in the form of a power series $\sum a_n x^n$ by solving $y' = \frac{1}{\sqrt{1-x^2}}$ in two ways. Deduce that $\frac{\pi}{6} = \frac{1}{2} + \frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{2^3} + \frac{1}{2}\cdot\frac{3}{4}\cdot\frac{1}{5}\cdot\frac{1}{2^5} + \cdots$

b) Verify that the equation $y'' + y' - x\,y = 0$ has a three term recursion formula and find it's solutions $y_1(x)$ and $y_2(x)$ such that $y_1(0) = 1$, $y_1'(0) = 0$, $y_2(0) = 0$, $y_2'(0) = 1$

UNIT III

5) a) Show that i) $\frac{d}{dx}\left[x^p J_p(x)\right] = x^p J_{p-1}(x)$ ii) $\frac{d}{dx}\left[x^{-p} J_p(x)\right] = -x^{-p} J_{p+1}(x)$

b) If λ_m and λ_n are distinct positive zeros of the Bessel function $J_p(x)$ .Show that

$$\int_0^1 x\, J_p(\lambda_m x) J_p(\lambda_n x)\,dx = 0 \quad \text{if } m \neq n$$
$$= \frac{1}{2} J_{p+1}(\lambda_n)^2 \quad \text{if } m = n$$

6) a)If the function f(x) has the Legendre series $f(x) = \sum_{n=0}^{\infty} a_n\, p_n(x)$ show that

$a_n = \frac{2n+1}{2}\int_{-1}^{1} f(x)\, p_n(x)\,dx$, where $p_n(x)$ is the Legendre polynomial

b) Find the first three terms of the Legendre series of $f(x) = 0$ if $-1 \leq x < 0$
$= x$ if $0 \leq x \leq 1$

UNIT IV

7) a) If the homogeneous system $\begin{cases} \frac{dx}{dt} = a_1(t)x + b_1(t)y \\ \frac{dy}{dt} = a_2(t)x + b_2(t)y \end{cases}$ has two solutions $\begin{cases} x = x_1(t) \\ y = y_1(t) \end{cases}$

and $\begin{cases} x = x_2(t) \\ y = y_2(t) \end{cases}$ on [a,b] show that

$\begin{matrix} x = c_1x_1(t) + c_2x_2(t) \\ y = c_1y_1(t) + c_2y_2(t) \end{matrix}$ is also a solution on [a,b] for any constant c_1 and c_2

b) show that $\begin{cases} x = 2e^{4t} \\ y = 3e^{4t} \end{cases}$ and $\begin{cases} x = e^{-t} \\ y = -e^{-t} \end{cases}$ are solutions of the homogeneous system

$$\begin{cases} \frac{dx}{dt} = x + 2y \\ \frac{dy}{dt} = 3x + 2y \end{cases}$$

8) a) Find the general solution of $\begin{cases} \frac{dx}{dt} = 7x + 6y \\ \frac{dy}{dt} = 2x + 6y \end{cases}$

b) Show that the wronskien of the two solutions $\begin{cases} x = e^{at}(A_1 \cos bt - A_2 \sin bt) \\ y = e^{at}(B_1 cosbt - B_2 sinbt) \end{cases}$ and

$\begin{cases} x = e^{at}(A_1 \sin bt + A_2 \cos bt) \\ y = e^{at}(B_1 sinbt + B_2 cosbt) \end{cases}$ is given by $w(t) = (A_1 B_2 - A_2 B_1)e^{2at}$

UNIT V

9) a) Determine the nature and stability of the critical point (0 , 0) of the linear

autonomous system $\begin{cases} \frac{dx}{dt} = 4x - 3y \\ \frac{dy}{dt} = 8x - 6y \end{cases}$

b) Show that (0,0) is an asymptotically stable critical point for the system

$$\begin{cases} \frac{dx}{dt} = -2x + xy^3 \\ \frac{dy}{dt} = -x^2y^2 - y^3 \end{cases}$$

10) a) Show that the function $E(x,y)=ax^2 + bxy + cy^2$ is positive definite if and only if $a > 0$ and $b^2 - 4ac < 0$ and is negative definite if and only if $a<0$ and $b^2 -4ac<0$

b) Show that (0,0) is unstable critical point for the system $\begin{cases} \frac{dx}{dt} = 2xy + x^3 \\ \frac{dy}{dt} = -x^2 + y^5 \end{cases}$ by

using Liapunov's direct method

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First Semester MSc.(Mathematics) Model Question Paper

(CCSS-2015 Admission)

Kannur University

MAT C 104 : REAL ANALYSIS

Time: 3 Hours **Max. Marks : 60**

(Each question carries 12 marks .Answer any **ONE** question from each unit)

UNIT I

1.a) If f and g are continuous real function on [a, b] which are differentiable in (a ,b), then show that there is a point x ∈ (a, b) at which

$[f(b) - f(a)]g'(x) = [g(b) - g(a)]f'(x)$. (4)

b)What happens in the above theorem if you take g(x) = x At that case is this theorem is applicable for complex valued function.?Justify your answer. (4)

c) Suppose i) f is continuous for $x \geq 0$,ii) f' exists for $x > 0$, iii) f' is monotonically increasing and iv) $f(0) = 0$

put $g(x) = \frac{f(x)}{x}$ $(x > 0)$,then prove that g is monotonically increasing. (4)

2. a) State and prove L-Hospital's rule. (8)

b) Let f be defined on [a,b];if f has a local maximum at a point x ϵ (a,b), and if $f'(x)$ exists , then prove that $f'(x) = 0$. (4)

UNIT II

3.a) State and prove Riemann- Stieltjes criterion for integrability. (6)

b) a) Assume that α increases monotonically and $\alpha' \in \mathcal{R}$ on [a, b]. Let f be a bounded real function on [a, b] Then show that $f \in \mathcal{R}(\alpha)$ if and only if $f\alpha' \in \mathcal{R}$ and

$\int_a^b f d\alpha = \int_a^b f(x)\alpha'(x)\, dx$. (6)

4.a)If f is continuous and non-negative on [a,b] then show that $\int_a^b f(x)dx \geq 0$. (2)

b)If $f_1 \in \mathcal{R}(\alpha)$ and $f_2 \in \mathcal{R}(\alpha)$ on [a, b] then show that $f_1 + f_2 \in \mathcal{R}(\alpha)$ and

$\int_a^b (f_1 + f_2) d\alpha = \int_a^b f_1 d\alpha + \int_a^b f_2 d\alpha$. (5)

c) If γ' ,the derivative of γ ,is continuous on [a, b] ,then prove that γ is rectifiable and

$\Lambda(\gamma) = \int_a^b |\gamma'(t)|\, dt$. (5)

UNIT III

5.a) Define uniform convergence of sequence of functions . How is it different from point wise convergence ? Illustrate with an example? (4)

b)Suppose K is compact and i)$\{f_n\}$ is a sequence of continuous function on K.

ii) $\{f_n\}$ converges pointwise to a continuous function f on K.

iii) $f_n(x) \geq f_{n+1}(x)$ *for all* $x \epsilon K$ n=1,2,3, ...

Then prove that $f_n \to f$ uniformly on K. (5)

c).Suppose $\lim_{n\to\infty} f_n(x) = f(x)$, $M_n = \sup | f_n(x) - f(x) |$ .Then show that $f_n \to f$ uniformly on E if and only if $M_n \to 0$ as $n\to \infty$. (3)

6.a)Suppose $f_n \to f$ uniformly on a set E in a metric space . Let x be a limit point of E and suppose that $\lim_{t\to x} f_n(t) = A_n$,n=1,2,3,... Then prove that $\{A_n\}$ converges and $\lim_{t\to x} f(x) = \lim_{n\to\infty} A_n$. (5)

b) c)Suppose $\{f_n\}$, $\{g_n\}$ are defined on E and

i)$\sum f_n$ has uniformly bounded partial sum

ii) $g_n \to 0$ uniformly on E

iii) $g_1(x) \geq g_2(x) \geq g_3(x) \geq \cdots$ for every $x \in E$

Prove that $\sum f_n g_n$ coverges uniformly on E. (4)

c) Suppose $\{ f_n \}$ is a sequence of functions defined on E and suppose $|f_n| \leq M_n , x \in E, n =$ 1,2,3, ...Then$\sum f_n$ converges uniformly on E if $\sum M_n$ converges (3)

UNIT IV

7. a) State and prove Stone-Weierstrass Theorem . (10)

b) Let A be a family of functions defined on a set K. When A is said to separate points on K and A vanishes at no point of K . (2)

8. a) If K is compact , if $f_n \in C(K)$ for n= 1,2,3,...... and if $\{ f_n\}$ is point wise bounded and equicontinuous on K then prove that $\{f_n \}$ is uniformly bounded on K. (4)

b)Prove that if K is a compact metric space , if $f_n \in C(K)$ for n=1,2,3,... and if $\{f_n\}$ converges uniformly on K then show that $\{f_n \}$ is equicontinuous on K (4)

c) Suppose A is an algebra of functions on a set E , A separates points on E, and A vanishes at no point of E. Suppose x_1, x_2 are distinct points of E and c_1, c_2 are constants , then prove that A contains a function f such that $f(x_1) = c_1$; $f(x_2) = c_2$. (4)

UNIT V

9. a)Define $E(z) = \sum_{n=0}^{\infty} \frac{z^n}{n!}$,$C(x) = \frac{1}{2}[E(ix)+E(-ix)]$ and $S(x) = \frac{1}{2i}[E(ix)-E(-ix)]$ .Then Prove that the function E is periodic ,with period $2\pi i$ and the functions C and S areperiodic,with period 2π. (5)

b)State and prove Parseval's theorem . (7)

10.a) b) Define gamma function and prove :If x > 0 and y > 0 , prove that

$$\int_0^1 t^{x-1} (1-t)^{y-1} dt = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)} \quad . (8)$$

b)Suppose $\sum C_n$ converges. Put $f(x) = \sum_{n=0}^{\infty} C_n x^n$ $(-1 < x < 1)$.Then $\lim_{x\to 1} f(x) = \sum_{n=0}^{\infty} C_n$.(4)

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First Semester MSc.(Mathematics) Model Question Paper
(CCSS-2015 Admission)
Kannur University
MAT C 105 : TOPOLOGY

Time: 3 Hours.
Max. Marks : 60

(Each question carries 12 marks .Answer any **ONE** question from each unit)

UNIT I

1. a) State Axiom of Choice and Maximum Principle. Define the product of sets $\{S_w\}_{w\in W}$. Justify the definition (4)

b) Let (**S,** χ) be a topological space and $A \subset S$. Then show that if A is open in S, then every open set in A is open in S and if A is closed in S, then every closed set in A is closed in S. (3)

c) State finite intersection property.Prove that a topological space is compact if and only if it has finite intersection property. (5)

2. a) When are two topologies said to be equivalent? When are the bases for two topologies equivalent? If S is a set and B_1, B_2 bases for topologies on S, then state and prove conditions for B_1, B_2 to be equivalent. (7)

b) If S is a compact topological space, then show that every infinite subset of S has a limit Point. (5)

UNIT II

3. a) If S and T are topological spaces, let f: $S \to T$ is continuous and surjective . If S is compact, then show that T is also compact (3)

b) Define homeomorphism. (2)

c) Let S be a set, and $\boldsymbol{W}\subset 2^{\mathbf{S}}$. Then show that there exists a weakest topology $\boldsymbol{U}$ on S such that $\boldsymbol{W}\subset\boldsymbol{U}$ (4)

d) Let f: $S \to R^{\mathbf{1}}$ is continuous . If S is compact, then show that f assumes its maximum and minimum on S. (3)

4. a) If S and T are topological spaces, let f: $S \to T$ is continuous and surjective . If S is connected, then show that T is also connected (3)

b) State and prove Tychnonoff theorem. (9)

UNIT III

5. a) Let C_1 and C_2 be disjoint compact subsets of a Hausdorff space S . Prove that there exists disjoint open sets U_1 and U_2 such that $C_1 \subset U_1$ and $C_2 \subset U_2$ (5)

b) Define a normal space. Let (S, ρ) be a metric space. Prove that S, with the associated metric topology is normal (7)

6. a) State and prove Tietze extension theorem. (5)

b) Define a completely regular space .Show that every subset of a completely regular space is completely regular. (4)

c) Prove that every locally compact Hausdorff space is completely regular (3)

UNIT 4

7. a) Prove that if S is a complete and totally bounded metric space then it is sequentially compact. Also prove that if S is complete, totally bounded and sequentially compact then S is compact. (7)

b) State and prove Baire category theorem (4)

c) Define Banach space. (1)

8. a) Define normed linear space. (2)

b) Let (S, ρ) be a complete metric space. Suppose $\{U_n\}$ is a countable collection of open sets each of which is dense in S. Prove that $\bigcap_{n=1}^{\infty} U_n \neq \varphi$ (5)

c) State and prove uniform boundedness principle. (5)

UNIT V

9. a) Define convergence of nets. Show that if A is a subset of a space X and if x ϵ X, then show that x ϵ A if and only if there exists a net in A which converges to x in X. (6)

b) Show that if τ_1,τ_2 are topologies on a set X such that a net in X converges to a point w.r.t. τ_1 if and only if it does so w.r.t. τ_2, then $\tau_1=\tau_2$. (6)

10. a) Define an image filter. Let X, Y be topological spaces, x∈ X and f : X→Y a function.Then show that f is continuous at x if and only if whenever a filter $\boldsymbol{F}$ converges to x in X ,then image filter $f_{\#}(\boldsymbol{F})$ converges to f(x) in Y (8)

b) Let X, Y be sets , f : X→ Y a function and $\boldsymbol{F}$ a filter on X. Prove that the family $f(\boldsymbol{F}) = \{f(A) : A\in \boldsymbol{F}\}$ is a base for a filter on Y. (4)

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Second Semester MSc.(Mathematics) Model Question Paper
(CCSS-2015 Admission)
Kannur University
MAT C 201 COMPLEX ANALYSIS

Time: 3 Hours. **Max. Marks** : 60

(Each question carries 12 marks .Answer any **ONE** question from each unit)

UNIT I

1. a) Construct linear transformation w = S(z) that maps the points (2,i,-2) onto the points (1,i,-1) respectively. (4)
   b) If f(z) is analytic and $f'(z) \neq 0$,at each point z of domain , then prove that the mapping is conformal (4)
   c) State and prove symmetry principle. (4)
2. a) Prove that the cross ratio (z_1 , z_2, z_3, z_4) is real if and only if the 4 points lie on a circle or a straight-line. (6)
   b) Describe the Riemann surface associated with the function w = z^n, n>1 is an integer. (6)

UNIT II

3. a) Derive Cauchy's integral formula . (5)
   b) If the piecewise differentiable closed curve γ doesnot pass through the point 'a' ,then prove that value of the integral $\int_\gamma \frac{dz}{z-a}$ is a multiple of 2πi . (4)
   c) Compute : $\int_c \frac{\sin \pi z^2+\cos \pi z^2}{(z-1)(z-2)}$ c: |z|=3 (3)
4. a) Suppose that is $\varphi(\zeta)$ continuous on the arc γ , then prove that the function $F_n(z) = \int_\gamma \frac{\varphi(\zeta)d\zeta}{(\zeta-z)^n}$ is analytic in each of the region determined by γ and its derivative is $F_n{}'(z) = n\, F_{n+1}(z)$. (5)
   b) State and prove Liouvillie's theorem. (4)
   c) Evaluate $\int_c \frac{z^3 dz}{(2z+i)^3}$ where c is the unit circle. (3)

UNIT III

5. a) If f(z) is analytic in a region Ω ,containing a point 'a' then prove that it is possible to write f(z)=f(a)+$\frac{f'(a)}{1!}(z-a)$+$\frac{f''(a)}{2!}(z-a)^2 + \cdots$ $+\frac{f^{(n-1)}(a)}{(n-1)!}(z-a)^{n-1} + f_n(z)(z-a)^n$, Where $f_n(z)$ is analytic in Ω (5)
   b) Define different types of singularities with example. (7)
6. a) State and prove Maximum principle. (4)
   b) If f(z) is defined and continuous on a closed bounded set E and analytic on the interior of E, then prove that the maximum of $|f(z)|$ on E is assumed on the boundary of E (4)
   c) State and prove Schwarz lemma. (4)

UNIT IV

7. a) Define residue of $f(z)$ at a point a. (2)

b) If $pdx + qdy$ is locally exact in Ω,then prove that $\int_\gamma pdx + qdy = 0$ for every cycle $\gamma \sim 0$ in Ω. (6)

c) How many roots of the equation $z^4 - 6z + 3 = 0$ have their modulus between 1 and 2 ? (4)

8. a) How many roots does the equation $z^7 - 2z^5 + 6z^3 - z + 1 = 0$ have in the disk $|z| < 1$. (4)

b) Prove that a region Ω is simply connected iff n(γ,a)=0 for all cycles γ in Ω and all points a which do not belong to Ω. (4)

c) Find $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx$. (4)

UNIT V

9. a) Prove that the arithmetic mean of a harmonic function over concentric circles $|z| = r$ is a linear combination of $\log r$, $\frac{1}{2\pi}\int_{|z|=r} u\, d\theta = \propto \log r + \beta$, and if u is harmonic in a disk then $\propto = 0$ and the arithmetic mean is constant . (6)

b) Expand $\frac{1}{z^2-3z+2}$ in the region i) $1 < |z| < 2$ ii) $|z| > 2$. (6)

10. a) Suppose that $f_n(z)$ is analytic in the region Ω_n , and that the sequence $\{f_n(z)\}$ converges to a limit function f(z) in a region Ω , uniformly on every compact subset of Ω . Then prove that $f(z)$ is analytic in Ω and $f_n'(z)$ converges uniformly to $f'(z)$ on every compact subset of Ω. (5)

b) Obtain the Taylor series expansion of $f(z) = \frac{z-1}{z+1}$ about z = 1. (3)

c) If the functions $f_n(z)$ are analytic and $\neq 0$ in a region Ω , and if $f_n(z)$ converges to f(z) , uniformly on every compact subset of Ω , then prove that $f(z)$ is either identically zero or never equal to zero in Ω . (4)

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Second Semester MSc.(Mathematics) Model Question Paper
(CCSS-2015 Admission)
Kannur University
MAT C 202: Functional Analysis

Time: 3 Hours **Max. Marks** : 60

(Each question carries 12 marks. Answer any **ONE** question from each unit)

UNIT-I

1	(a)	Let Y be a subspace of a normed space X. show that Y and its closure are normed spaces with the induced norm.	2
	(b)	Define quotient norm, If Y is a closed subspace of a normed space X show that X / Y is a normed space with the quotient norm.	5
	(c)	State and prove the Riesz Lemma.	5
2	(a)	State and prove Hahn-Banach Separation theorem.	7
	(b)	If $E^0 \neq \phi$ and b belongs to the boundary of E in X , show that there is nonzero f $\epsilon X'$ such that Ref(x) ≤ Ref(b) for all x $\epsilon \bar{E}$	5

UNIT-II

3	(a)	Let X be a normed space.Show that a subset E of X is bounded if and only if $f(E)$ is bounded for every f $\epsilon X'$	5
	(b)	State and prove Closed graph theorem.	7
4	(a)	Let X be a normed space and $P: X \rightarrow X$ be a projection. Show that P is closed if and only if $R(P)$ and $Z(P)$ are closed in X	6
	(b)	Let X and Y be normed spaces. If Z is a closed subspace of X, then show that the quotient map Q from X to X/Z is continuous and open	6

UNIT-III

5	(a)	Let X be a normed space.Show that if X' is separable then X is separable.	4
	(b)	Show that for $1 \leq p < \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$, $(l^p)'$ is linearly isometric to l^q	5
	(c)	Define Transpose of a bounded linear map. Let X,Y be normed spaces and F∈BL(X,Y) Show that $\|F\| = \|F'\| = \|F''\|$	3
6	(a)	Let 1≤ p≤ ∞ and $\frac{1}{p} + \frac{1}{q} = 1$ Prove that the dual of C_{00} with $\| \quad \|_p$ is linearly isometric to l^q	6
	(b)	Let X, Y be Banach spaces, F∈BL(X, Y). If R(F)=Y, then prove that F' is bounded below.	3
	(c)	Let X, Y be normed spaces. If F∈BL(X,Y) show that $\|F\| = \text{Sup}\{\|y'(F(x))\|: x \in X,\ \|x\| \leq 1, y' \in Y', \|y'\| \leq 1\}$	3

UNIT-IV

7	(a)	If X is reflexive, show that X' is reflexive.	5
	(b)	Show that a closed subspace of a reflexive space is reflexive.	5
	(c)	Let X be a reflexive normed space. Show that X' is separable if X is separable.	2

8 (a) If X is a non−zero reflexive space show that every non−empty closed convex subset of X contains an element of minimal norm. 6

(b) Let X be a Banach spaces which is uniformly convex in some equivalent norm. Show that X is reflexive. 6

UNIT-V

9 (a) Define inner product space. Give one example. 3

(b) State and prove Parallelogram Law. 3

(c) State and prove Schwarz − inequality for inner product. 6

10 (a) Describe the Gram− Schmidt orthonormalization process. 7

(b) If $\{u_\alpha\}$ is an orthonormal basis in a Hilbert Space H, Show that for every x$\in X$, $\|x\|^2 = \sum_n |< x, u_n >|^2$ $where$ $\{u_{1,}u_{2,} \dots\} = \{u_\alpha : < x, u_\alpha > \neq 0\}$ 5

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Second Semester MSc.(Mathematics) Model Question Paper

(CCSS-2015 Admission)

Kannur University

MAT C 203 : ALGEBRA II

Time: 3 Hours. **Max. Marks** : 60

(Each question carries 12 marks .Answer any **ONE** question from each unit)

UNIT I

1) a)State and prove Gauss's lemma (5)
b)Prove that every PID is a UFD (4)
c) Define UFD.Give an example with expalanations (3)

2) a) Show that $\mathbb{Z}[i]$ is a Euclidean domain (6)
b)State and prove Euclidean Algorithm (6)

UNIT II

3) a)State and prove Kronecker's theorem. (5)
b) Prove that Q($2^{1/2}, 2^{1/3}$)=Q($2^{1/6}$). (5)
c)Define algebraically closed field and give one example. (2)

4) a) Prove that if E is a finite extension field of F and K is a finite extension of E,then K is a finite extension of F and [K:F]=[K:E][E:F]. (5)
b)Prove that if α and β are constructible real numbers then so are $\alpha + \beta$, $\alpha - \beta, \alpha\beta$ and α/β if $\beta \neq 0$ (5)
c)Define simple extension. (2)

UNIT III

5) a)State and prove conjugation isomorphism theorem. (5)
b)If F is any finite field, then prove that for every positive integer n, there is an irreducible polynomial in F[x] of degree n (4)
c) Let F be a finite field of characteristic p.Then prove that the map $\sigma_p : F \to F$ defined by $\sigma_p(a) = a^p$ for a∈F is an automorphism of F.Also $F_{\sigma_p} \cong Z_p$. (3)

6) a) State isomorphism extension theorem. (8)
b)Define primitive n^{th} roots of unity. Find the number of primitive n^{th} roots of unity in GF(9). (4)

UNIT IV

7) a) Prove that a field E,where $F \leq E \leq \bar{F}$ is a splitting field over F iff every automorphism of $\bar{F}$ leaving F fixed maps E onto itself and thus induces an automorphism of E leaving F fixed. (8)
b)Prove that every field of characteristic zero is perfect. (4)

8) a)State and prove primitive element theorem. (6)
b)Prove that if E is a finite extension of F ,then E is totally inseparable over F iff each $\propto$ in E , $\propto \neq$F, is totally inseparable. (4)
c)Define splitting field over a field F. (2)

UNIT V

9) a)Let F be a field of characteristic 0,and let $a \in F$.If K is the splitting field of x^n-a over F, then prove that G(K/F) is a solvable group. (7)
b)Define n-th cyclotomic polynomial over F. (2)
c)Find |G(K/$\mathbb{Q}$)| where K= $\mathbb{Q}(\sqrt{2}, \sqrt{3}, \sqrt{5})$. (3)

10) a)Define symmetric function over a field F. (2)
b)Let K be a finite normal extension of a field F , with Galois group G(K/F).For a field E,where F≤E≤K,let λ(E) be the subgroup of G(K/F) leaving E fixed.Then prove that for H≤G(K/F) , $\lambda(K_H)$=H. (6)
c)Prove that the Galois group of the p^{th} cyclotomic extension of $\mathbb{Q}$ for a prime p is a cyclic group of order p-1 (4)

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Second Semester MSc.(Mathematics) Model Question Paper

(CCSS-2014 Admission)

Kannur University

MAT C 204 Differential Equations II

Time: 3 Hours **Max. Marks**: 60

(Each question carries 12 marks. Answer any **ONE** question from each unit)

UNIT-I

1 (a) Verify that the equation (6x+yz)dx+(xz-2y)dy+(xy+2z)dz=0 is integrable and hence solve it. 5

(b) Find the complete integral of the equation $p^2x + q^2y = z$ by using Charpit's method. 5

(c) Form the p.d.e. for the parametric family of surface $(x-a)^2 + (y-b)^2 + z^2 = 1$. 2

2 (a) If h_1=0 and h_2=0 are compatible with f=0 then $h_1$and h_2 satisfy $\frac{\partial(f,h)}{\partial(x,u_x)} + \frac{\partial(f,h)}{\partial(y,u_y)} + \frac{\partial(f,h)}{\partial(z,u_z)}$=0. 5

(b) Show that $(x-a)^2 + (y-b)^2 + z^2 = 1$ is the complete integral of $z^2(p^2+q^2+1) = 1$. 2

(c) Solve by Jacobi's method the equation $z^2 + zu_z - {u_x}^2 - {u_y}^2 = 0$. 5

UNIT-II

3 (a) Find the Monge cone of $p^2 + q^2 = 1$ at (0,0,0). 4

(b) Find the solution of the equation $z=\frac{1}{2}(p^2+q^2) + (p-x)(q-y)$ which passes through the x- axis. 8

4 (a) Solve the Cauchy problem $zz_x + z_y = 1$ for the initial data curve C: x_0= s, y_0=s, $z_0=\frac{s}{2}$ and 0≤ s ≤1 . 5

(b) Find by the method of characteristics, the integral surface of pq = xy which passes through the curve z=x, y=0. 7

UNIT-III

5 (a) Find the solution of vibration of string by using method of separation of variables. 6

(b) Solve one dimensional wave equation $y_{tt} - c^2 y_{xx}$=0 ,0<x<l ,t>0, y(x,0)=$f(x)$ *and* $y_t(x,0) = g(x)$, 0≤x≤l, y(0,t)=y(l,t)=0, t>0. 6

6 (a) Reduce the equation $4u_{xx} - 4u_{xy} + 5u_{yy} = 0$ to canonical form. 5

(b) Solve $y_{tt} = 4\, y_{xx}$, y(0,t)=y(5,t)=0 and y(x,0)=0 , $y_t(x,0) = x^2$. 5

(c) State Green's theorem. 2

UNIT-IV

7 (a) Derive Poisson integral formula for the Dirichlet's problem for a circle. 6

(b) State and prove the Dirichlet's problem for a Rectangle. 6

8	(a)	State minimum principles.	2
	(b)	State and prove the Neumann problem for a Circle.	6
	(c)	If u(x, y) is harmonic in a bounded domain D and continuous in $\overline{D}$=D ∪ B, then show that u attains its maximum on the boundary B of D.	4

UNIT-V

9	(a)	Using Duhamel's principle, solve $u_{tt} - c^2 u_{xx} = F(x,t)$, $-\infty < x < \infty,\ t > 0$ $u(x, 0) = u_t(x,0) = 0$, $-\infty < x < \infty$	6
	(b)	Find solution of Laplace equation in spherical polar co-ordinates using the method of separation of variables.	6
10	(a)	State and prove Kelvin's inversion theorem.	6
	(b)	Define family of equipotential surfaces. Find the general form of the potential function for the family of surfaces , $x^2+y^2+z^2 = c$.	6

Name : ……………..
Reg. No. : ……………

Second Semester MSc.(Mathematics) Model Question Paper
(CCSS-2015 Admission)
Kannur University
MAT C 205 : MEASURE AND INTEGRATION

Time: 3 Hours. **Max. Marks** : 60

(Each question carries 12 marks .Answer any **ONE** question from each unit)

UNIT I

1. a) Define a σ- algebra and give one example. (2)
   b) If f and g are two measurable real valued functions and c is a real number, then show that the functions i) cf ii) f + g iii) fg and iv) |f | are also measurable (6)
   c) Let μ be a measure defined on a σ- algebra **X** . If $\{E_n\}$ is an decreasing sequence in **X**, and $\mu(E_1) < +\infty$. Then show that , $\mu(\cap_{n=1}^{\infty} E_n) = \lim \mu(E_n)$. (4)

2. a)Define measure. (2)
   b)If μ be a measure defined on a σ- algebra **X** and A is a fixed set in **X** , then show that the function λ ,defined for $E \in$ **X** ,by $\lambda(E) = \mu(A \cap E)$ is a measure on **X.** (3)
   c) If f is a measurable real valued function, then show that f^2 and | f | are measurable . (3)
   d) If $\{f_n\}$ is a sequence in M(X, **X**), $f(x) = \inf f_n(x)$, $F(x)= \sup f_n(x)$, $f^*(x) = \lim\inf f_n(x)$ and $F^*(x) = \lim\sup f_n(x)$, then show that f, F, f^* and $F^* \in$ M(X, **X**). (4)

UNIT II

3. a) Define the integral of a simple function with respect to μ. If φ and ψ be two simple functions in M^+(X , **X**) then show that $\int(\varphi + \psi)\, d\mu = \int \varphi d\mu + \int \psi\, d\mu$. (5)
   b) Define L_p space for $1 \le p < +\infty$. (2)
   c) If f and h belong to L_p , p >1 and 1/p+1/q=1. Then show that f+h belongs to L_p and $\|f+h\|_p \le \|f\|_p + \|h\|_p$ (5)

4. a)State and prove Monotone convergence theorem. (6)
   b) Prove that a measurable function f belongs to L if and only if | f | belongs to L (4)
   c)State Minkowski Inequality. (2)

UNIT III

5. a)Let (f_n) be a sequence in L_p which converges almost everywhere to a measurable function f. if there exists a g in L_p such that $|f_n(x)| \le g(x)$, $x \in X$, $n \in N$ then show that, f belongs to L_p and (f_n) convergence in L_p to f. (6)
   b) Let $\mu(X) < +\infty$, and let (f_n) be a sequence in L_p which converges almost everywhere to a measurable function f. If there exists a constant K such that $|f_n(x)| \le K$, $x \in X$, $n \in N$ then show that, f belongs to L_p and (f_n) convergence in L_p to f. (3)
   c) Define almost uniform convergence. Show that uniform convergence implies almost uniform convergence. (3)

6. a) State and prove Egoroff's theorem . (7)

b) Define convergence in L_p and convergence almost everywhere .Is Convergence in L_p imply convergence almost every where? Justify your answer. (5)

UNIT IV

7. a) Define positive and negative set with respect to a charge $\boldsymbol{\lambda}$. (2)
   b) If $\boldsymbol{\lambda}$ is a charge on **X,** then show that there exists sets P and N in **X** with X = P∪N, P∩N = φ, and such that P is positive and N is negative with respect to $\boldsymbol{\lambda}$. (6)
   c) Prove that if μ be a σ-finite measure defined on an algebra $\boldsymbol{A}$,then there exists a unique extension of μ to a measure on μ^* . (4)

8 a) Let $\boldsymbol{A}$ be an algebra on a set X, μ be a measure defined on $\boldsymbol{A}$ .Define the outer measure μ^* generated by μ. (2)
   b) Define μ^* measurable sets (2)
   c) Show that the collection $\boldsymbol{A}^*$ of all μ^* - measurable sets is a σ- algebra containing $\boldsymbol{A}$. Also prove that If (E_n) is a disjoint sequence in $\boldsymbol{A}^*$, then $\mu^*(\cup E_n) = \sum_{n=1}^{\infty} \mu^*(E_n)$. (8)

UNIT V

9. a) State and prove Monotone Class Lemma. (6)
   b) Show that (6)
   i) If E is a measurable subset of Z , then every section of E is measurable .
   ii) If f is a measurable function on Z to R, then show that every section of f is measurable.

10. a) Let (X, χ, μ), (Y, γ, ϑ) be σ-finite measure spaces. If E ∈ $\boldsymbol{Z} = \chi \times \gamma$, then prove that the functions defined by $f(x) = \vartheta(E_x)$, $g(y) = \mu(E^y)$ are measurable and (6)

$$\int_X f d\mu = \pi(E) = \int_Y g d\mu.$$

   b) Let (X, χ, μ) be the measure space on the natural numbers X = N with the counting measure defined on all subsets of X. Let (Y, γ, ϑ) be an arbitrary measure space. Show that a set E in Z= XxY belongs to $\boldsymbol{Z} = \chi \times \gamma$ if and only if each section E_n of E belongs to γ .In this case show that there is a unique product measure π and $\pi(E) = \sum_{n=1}^{\infty} \vartheta(E_n),\ E \in \boldsymbol{Z}$. (6)

Third Semester MSc.(Mathematics) Model Question Paper
(CCSS-2015 Admission)
Kannur University
MAT C 301 :DIFFERENTIAL GEOMETRY

Time: 3 Hours. **Max. Marks** : 60

(Each question carries 12 marks .Answer any **ONE** question from each unit)

UNIT I

1. a) Find the level sets and graphs of a function f:U→ℝ (where U subset of $\mathbb{R}^{n+1}$) defined by $f(x_1, \ldots, x_{n+1}) = x_1^2 + \ldots + x_{n+1}^2$.
   b) Define level sets and graph of a function f:U→ℝ .
   c) Show that the graph of any function $f:\mathbb{R}^n \to \mathbb{R}$ is the level set for some function $F:\mathbb{R}^{n+1} \to \mathbb{R}$.
2. a) Define an integral curve.
   b) Let χ be a smooth vector field on an open set U subset of $\mathbb{R}^{n+1}$ and let p ϵ U. Then prove that there exist an open interval I containing 0 and an integral curve $\alpha:I \to U$ of χ such that α(0) = p and If $\beta:\tilde{I} \to U$ is any other integral curve of χ with β(0) = p, then $\tilde{I} \subset I$ and β(t) = α(t) for every t ϵ $\tilde{I}$.
   c) Find an integral curve through p = (1,0) of the vector field χ(p) = (p, X(p)),where $X(x_1, x_2) = (-x_2, x_1)$.

UNIT II

3. a) Write a note on curvature of plane curves.
   b) Prove that local parametrization of a plane curve exists and is unique upto reparametrization.
   c) Let C be the circle $f^{-1}(r^2)$,where $f(x_1, x_2)=(x_1 - a)^2+(x_2 - b)^2$ oriented by the outward normal $\frac{\nabla f}{\|\nabla f\|}$. Find the curvature of C at pϵC.
4. a) Define Euclidean parallel . Prove that a vector field χ along α is Euclidean parallel along α iff $\dot{\chi} = 0$. (3)

   b) Define Weingarten map. Prove that the Weingarten map L_p is self-adjoint.
   c) Prove that the function which sends v into $\nabla_v f$ is a linear map from $\mathbb{R}_p^{n+1} \to \mathbb{R}$.

UNIT III

5. a) Find the length of the parametrized curve $\alpha:[-1,1]\to\mathbb{R}^3$ defined by $\alpha(t)=(\cos 3t, \sin 3t, t)$.
   b) Let C be a connected oriented plane curve and let $\beta: I \to C$ be unit speed global parametrization of C. Then show that β is either periodic or one to one. Also show that β is periodic iff C is compact.
   c) Define length of a connected oriented plane curve. Let C be denote the circle $(x_1-a)^2+(x_2-b)^2=r^2$ oriented by its outward normal. Also let $\alpha:I \to C$ be defined by $\alpha(t)=(a+ r\cos 2t, b - r\sin 2t)$. Find length of C.
6. a) Let $\boldsymbol{\eta}$ denote the 1-form on $\mathbb{R}^2$-{0} defined by $\eta = -\frac{x_2}{x_1^2+x_2^2}dx_1 + \frac{x_1}{x_1^2+x_2^2}dx_2$ and let C denote the ellipse $\frac{x_1^2}{a^2}+\frac{x_2^2}{b^2}=1$, oriented by its inward normal. Check whether $\boldsymbol{\eta}$ is exact or not. If not how can be $\boldsymbol{\eta}$ made exact.
   b) Let $\boldsymbol{\eta}$ denote the 1-form on $\mathbb{R}^2$-{0} defined by $\eta = -\frac{x_2}{x_1^2+x_2^2}dx_1 + \frac{x_1}{x_1^2+x_2^2}dx_2$. Then for

α: [a,b]$\rightarrow \mathbb{R}^2$-{0} any closed piecewise smooth parametrized curve in $\mathbb{R}^2$-{0} show that $\int_\alpha \eta=2\pi$k for some integer k. .

UNIT IV

7. a) Define differential of a smooth map. Also show that it does not depend on the choice of the parametrized curve.
   b) Define co-ordinate vector fields along a smooth map φ.
   c) Let φ be the parametrized torus in $\mathbb{R}^3$ given by $\varphi(\theta,\emptyset)$=((a+bcos $\emptyset$)cos θ,(a+bcos $\emptyset$)sin θ,b sin $\emptyset$). Find Gaussian curvature.
8. a) Let $\emptyset$: U$\rightarrow \mathbb{R}^{n+1}$ be a parametrized n-surface in $\mathbb{R}^{n+1}$ and let pϵU. Then prove that there exists an open set $U_1 \subset V$ about p such that $\varphi(U_1)$ is an n-surface in $\mathbb{R}^{n+1}$.
   b) Let S be an n-surface in $\mathbb{R}^{n+1}$ and $\varphi: U \rightarrow \mathbb{R}^{n+1}$ be a parametrized n-surface and let pεU . Then does there exist any relation between the Weingarten map.
   c) Define an n-surface in $\mathbb{R}^{n+k}$ and give an example.

UNIT V

9. a) Define a rigid motion. Let ψ: $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ be defined by $\psi(x_1,x_2) = (x_1\cos\theta$-$x_2\sin\theta$, $x_2\cos\theta$+$x_1\sin\theta)$ $\theta\epsilon\mathbb{R}$. Prove that ψ is a rigid motion.
   b) Let ψ be a rigid motion of $\mathbb{R}^{n+1}$.Then prove that there exists a unique orthogonal transformation Ψ_1 and a unique orthogonal translation Ψ_2 such that $\psi = \Psi_2 o \Psi_1$.
   c) Let ψ: $\mathbb{R}^{n+1} \rightarrow \mathbb{R}^{n+1}$ be a rigid motion .Then prove that ψ is smooth and ψ maps $\mathbb{R}^{n+1}$ onto $\mathbb{R}^{n+1}$.
10 a) Define local isometry and isometry.
   b) Let S be the punctured plane $\mathbb{R}^2$ –{(0,0)} and $\tilde{S}$ be the cone $3{x_1}^2 + 3x_2^2 - x_3^2 = 0$ x_3 >0 in $\mathbb{R}^3$. Define ψ: S$\rightarrow\tilde{S}$ by $\psi(r\cos\theta, r\sin\theta) = (\frac{r}{2}\cos 2\theta, \frac{r}{2}\sin 2\theta, \frac{\sqrt{3}}{2}r)$ where $(r,\theta), r > 0$ are polar coordinates on $\mathbb{R}^2$ –{(0,0)} . Show that ψ is a local isometry.
   c) Let $\varphi: U \rightarrow \mathbb{R}^{n+k}$ be a parametrized n-surface and let E_i , $i\epsilon\{1,2,\dots,n\}$ be the coordinate vector fields along φ. Then prove that for pϵ U and $i,j\epsilon$ $\{1,2,\dots,n\}$ $(D_{E_i}$ $E_j)$. $E_k = \frac{1}{2}(\frac{\partial g_{ik}}{\partial x_j} + \frac{\partial g_{jk}}{\partial x_i} - \frac{\partial g_{ij}}{\partial x_k})$ where g_{ij}:U$\rightarrow\mathbb{R}$ are defined by $g_{ij} = E_i . E_j$.

Third Semester MSc.(Mathematics) Model Question Paper
(CCSS-2015 Admission)
Kannur University
MAT C 302 : Fuzzy Mathematics

Time: 3 Hours. **Max. Marks** : 60

(Each question carries 12 marks .Answer any **ONE** question from each unit)

UNIT I

1. a) If A is a fuzzy set on $\mathbb{R}$ show that A is convex iff $A(\lambda x_1+(1-\lambda)x_2)\geq\min[A(x_1), A(x_2)]$ for all $x_1,x_2 \in \mathbb{R}$ and all $\lambda\in[0,1]$ (4)
   b) State and prove first decomposition theorem and give an illustration of the theorem. (4)
   c) For $A\in \mathcal{F}(X)$ prove that ${}^{\alpha+}A = \bigcup_{\alpha<\beta} {}^{\beta}A = \bigcap_{\alpha<\beta} {}^{\beta+}A$ (4)

2. a) Define α- cuts and strong α- cuts. (2)
   b) Let f: X→Y be an arbitrary crisp function. Then prove that for any $A_i\in\mathcal{F}(X)$ and $B_i\in\mathcal{F}(Y)$,i∈I f fuzzyfied by the extension principle satisfies the following properties (i) $f(\bigcup_{i\in I} A_i) = \bigcup_{i\in I} f(A_i)$ (3)
   (ii)if $B_1 \subseteq B_2$ then $f^{-1}(B_1) \subseteq f^{-1}(B_2)$
   c) Let $A,B\in \mathcal{F}(X)$ and $\alpha\in[0,1]$ then prove that $A\subseteq B$ iff ${}^{\alpha}A\subseteq {}^{\alpha}B$ and $A\subseteq B$ if and only if ${}^{\alpha+}A\subseteq {}^{\alpha+}B$. (4)
   d) Explain the concepts of standard Union, intersection of fuzzy sets and the complement of a fuzzy set with the help of examples (3)

UNIT II

3. a) Define aggregation operations. (2)
   b) Let c: [0,1] ⟶[0,1] monotonically decreasing function which also involutive .Then prove that c is continuous and C also satisfies conditions c(0)=1 and c(1)=0. Moreover C must be bijective function (5)
   c) State and prove second characterisation theorem. (5)
4. a) Given a t-norm i and an involutive fuzzy complement c.Show that the binary operation u on [0,1] defined by u(a,b) =c(i(c(a),c(b))) for all a,b∈[0,1], is a t-conorm and < i,u,c > is a dual triple. (5)
   b) For all a,b∈[0,1], prove that $i_{min}(a,b) \leq i(a,b)\leq\min(a,b)$ (4)
   c) What is a t-conorm? . Give an example? (3)

UNIT III

5. a) Let $A\in\mathcal{F}(R)$. Then show that A is a fuzzy number iff there exists a closed interval $[a,b] \neq\Phi$ such that (6)

$$A(x) = \begin{cases} 1 \text{ for } x \in [a,b] \\ l(x) \text{for } x \in (-\infty,a) \\ r(x) \text{ for } x \in (b,\infty), \end{cases}$$

   where l is a function from (-∞,a) to [0,1] that is monotonic increasing and continuous from the right and such that l(x) =0 for $x\in(-\infty,\omega_1)$; r is a function from (b,∞), to [0,1] that is monotonic decreasing , continuous from the left and such that r(x)= 0 for $x\in(\omega_2,\infty)$
   b) Explain fuzzy model of group decision making proposed by Blin and Whinston (6)

6. a) Prove that MIN[A,MAX(B,C)] =MAX[MIN(A,B),MIN(A,C)] (6)

b) Solve the fuzzy linear programming problem maximize $z=.4x_1+.3x_2$ such that (6)

$x_1+x_2 \leq B_1$

$2x_1+x_2 \leq B_2$ and $x_1, x_2 \geq 0$

Where B_1 is defined by $B_1(x) = 1$ when $x \leq 400$

$= (500-x)/100$ when $400 \prec x \leq 500$

$= 0$, when $500 \prec x$ and B_2 is defined by

$B_2(x) = 1$ when $x \leq 500$

$= (600-x)/100$ when $500 \prec x \leq 600$

$= 0$, when $600 \prec x$

UNIT IV

7. a) What you mean by projection and cylindric extension of fuzzy relations? (4)

b) Let i be a continuous t-norm and $w_i(a,b)= \mathrm{Sup}\{ x \in [0,1] : i(a, x) \leq b \}$, then for $a,b, a_j \in [0,1]$ (6)

a) Show that $w_i[\inf_{j\in J} a_j ,b] \geq \sup_{j\in J} w_i(a_j, b)$

b) Show that $w_i[\sup_{j\in J} a_j ,b]= \inf_{j\in J} w_i(a_j, b)$

c) What is a fuzzy relation ? give one example (2)

8. a) The fuzzy binary relation R defined on sets X={1,2,…,100} and Y={50,51,…,100} and represent the relation " x is much smaller than y". It is defined by the membership function (4)

$R(x,y) = \{ 1-x/y$ for $x \leq y$ and 0 otherwise, where $x \in X$ and $y \in Y$ then

(i) What is the domain of R

(ii) What is the range of R

b) Define : Fuzzy partial ordering , Fuzzy homomorphism, (4)

c) Let i be a continuous t-norm and $w_i(a,b) = \mathrm{Sup}\{ x \in [0,1] : i(a, x) \leq b \}$, then for $a,b, d \in [0,1]$ (4)

i)Show that $w_i[i(a, b),d]= w_i[a,w_i (b, d)]$

ii)$a \leq b$ implies $w_i(a, d) \geq w_i (b, d)$ and $w_i(d, a) \leq w_i(d, b)$

iii) $w_i[w_i(a,b),b] \geq a$.

UNIT V

9. a) Describe fuzzy quantifiers of the two different kinds and illustrate with examples. (5)

b) Describe the generalised hypothetical syllogism. (3)

c) Explain Linguistic hedges (4)

10 a) Describe generalised modus tollens in terms of compositional rule. (2)

b) Let $X = \{x_1, x_2, x_3\}$, $Y = \{y_1, y_2\}$ and ***X*** and ***Y*** be variables taking values from the sets X and Y respectively . Assume that a proposition " If ***X*** is A, then***Y*** is (4)

B" is given where $A = .5/x_1 + 1/x_2 + .6/x_3$ and $B = 1/y_1 + .4/y_2$. Assume that a fact expressed by" *Y* is B' " is given where $B' = .9/y_1 + .7/y_2$ .Derive the conclusion ***X*** is A'.

c) Describe the classification into 4 types of simple fuzzy propositions. (6)

Name:……..
Reg.No.:………

Third Semester M.Sc.(Mathematics) Model Question Paper
(CCSS-2015 Admission)
Kannur University

MAT C 303 PROBABILITY

Time: 3 Hours Total Marks: 60

(Each question carries 12 marks. Answer any ***ONE*** question from each unit)

UNIT I

1. a) Define a probability space .Give one example. (3)
   b) Let $(\Omega, \boldsymbol{\mathcal{B}}, P)$ be a probability space. Show that for events $B_i \subset A_i$, $P(\cup_i A_i) - P(\cup_i B_i) \leq \sum_i P(A_i) - P(B_i)$. (3)
   c) State and prove first extension Theorem on probability measures. (6)

2. a) Define a λ-system.Is every σ-field is a λ-system?. (3)
   b) Let P_1, P_2 be two probability measures on $(\Omega, \boldsymbol{\mathcal{B}})$. Show that the class $L =\{A\in \boldsymbol{\mathcal{B}} : P_1(A)= P_2(A)\}$ is a σ-field. (3)
   c) Let A be a field and P is a probability measure on A. Let $\mathcal{G}$ be a class of non decreasing limits of A. Define a set function π: $\mathcal{G} \rightarrow [\mathbf{0}, \mathbf{1}]$ as $G = \lim_{n\to\infty} \uparrow B_n \epsilon \mathcal{G}$ where $B_n \epsilon A$, $\pi(G) = \lim_{n\to\infty} \uparrow P(B_n)$ .Then prove that π satisfies the following properties. (6)
      i) $\pi(G_1 \cup G_2) = \pi(G_1) + \pi(G_2) - \pi(G_1 \cap G_2)$.
      ii) If $G_n \in \mathcal{G}$ and $G_n \uparrow G$ then $G \in \mathcal{G}$ and $\pi(G) = \lim_{n\to\infty} \pi(G_n)$.

UNIT II

3. a) Define inverse map. Prove that the inverse maps preserves complements, unions and intersections . (4)
   b) Show that if $\boldsymbol{\mathcal{B}}'$ is a σ-field of subsets of Ω', then $X^{-1}(\boldsymbol{\mathcal{B}}')$ is a σ- field of subsets of Ω. (3)
   c) State and prove Kolmogorov Zero-One law. (5)

4. a) When are 2 events said to be independent? Extend it for finite number of events. (2)
   b) State and prove the basic criterion for independence of σ-fields. (5)
   c) Prove that if C' is a class of subsets of Ω' then $X^{-1}(\sigma(C')) = \sigma(X^{-1}(C'))$. (5)

UNIT III

5. a) Define simple function. Give one example . (2)

   b) State and prove measurability theorem. (4)

   c) Let P_1 be a probability measure on $\boldsymbol{B}_1$, and suppose K: Ω_1x$\boldsymbol{B}_2 \rightarrow [0,1]$ is a transition function. Prove that K and P_1 uniquely determine a probability on $\boldsymbol{B}_1$x$\boldsymbol{B}_2$ by the formula (6)

   $P(A_1xA_2)=\int_{A1} K(\omega_1, A_2)P_1(d\omega_1) for\ all\ A_1xA_2\ \in RECT$,where RECT is a class of all measureable rectangles.

6. a) Define expectation of simple functions . (2)

   b) Let X, Y be simple functions. Then Show that (2)

   $E(\alpha X + \beta Y) = \alpha E(X) + \beta E(Y)$for $\alpha, \beta \in \boldsymbol{R}$

   c) Suppose $X \in L_1$. Show that for any $\lambda>0$, $P[|X|\geq \lambda] \leq \lambda^{-1}E(|X|)$. (2)

   d) State and prove monotone convergence theorem. (6)

UNIT IV

7. a) State and prove Schwartz inequality.` (5)

   b) Define convergence in probability. (2)

   c) Show that $X_n \rightarrow X$ in probability iff each subsequence $\{X_{nk}\}$ contains a further subsequence $\{X_{nk(i)}\}$ which converges almost surely for X (5)

8. a) State and prove Holder's inequality (6)

   b) Prove that as $n\rightarrow \infty$,$X_n \rightarrow X$ in L_1 if and only if $sup_{A\in B} \left|\int_A\ XndP - \int_A\ XdP\right| \rightarrow 0$. (6)

UNIT V

9. a) Suppose the two sequences $\{X_n\}$ and $\{X_n'\}$ are tail equivalent. Show that, $\sum_n X_n$ converges almost surely iff $\sum_n X_n$' converges almost surely. (3)

   b) State and prove Skorohod's Inequality. (7)

   c) State Kolmogorov three series theorem (2)

10. a) Suppose $\{X_n, n \geq 1\}$ are independent random variables and define $S_n = \sum_{j=1}^{n} X_j$, (6)

    Prove that if $\sum_{j=1}^{n} P[|X_j| > n] \rightarrow 0$ and $\frac{1}{n^2}\sum_{j=1}^{n} EX_j{}^2\, I_{[|X_j|\leq n]} \rightarrow 0$.

    then $\frac{S_{n-a_n}}{n} \rightarrow 0$ in probability,where $a_n = \sum_{j=1}^{n} EX_j\, I_{[|X_j|\leq n]}$.

    b) State and prove Kolmogorov Convergence criterion. (6)

Third Semester M.Sc. Mathematics Model Question Paper
(CCSS-2015 Admission)
Kannur University
MAT E 304 : ADVANCED FUNCTIONAL ANALYSIS

'ime: Three hours Max: 60 marks

Each question carries 12 marks.

(Answer any ONE question from each unit)

Unit1

1. (a) Let X be a normed space and $A \in BL(X)$.Show that A is invertible if and only if A is bounded below and range of A is dense in X . [4]
   (b) If $k \in \sigma_e(A)$, Show that $| k |\leqslant inf_n \| A^n \|^{\frac{1}{n}} \leqslant \| A \|$. [4]
   (c) If X is a Banach space , show that the set of all invertible operators is open in $BL(X)$ and the map $A \to A^{-1}$ is continuous. [4]

2. (a) Let X be a Banach space and $A \in BL(X)$.Show that $\sigma(A)$ is a compact subset of K [4]
   (b) Let X be a normed space and Y be a Banach space . If $F_n \in CL(X,Y)$, for n=1,2,...,$F \in BL(X,Y)$ and $\|F_n - F\| \to 0$, show that $F \in CL(X,Y)$ [4]
   (c) Show that the inegral operator $F : L^p([a,b]) \to L^q([a,b]), \frac{1}{p}+\frac{1}{q}=1$ defined by $F(x)(s) = \int_a^b k(s,t)x(t)dm(t), x \in L^p([a,b]), s \in [a,b]$ where $k(,) \in L^q([a,b] \times [a,b])$ is compact [4]

Unit II

3. (a) Let X be a normed space over K and $A \in CL(X)$. Show that one of the following holds
      i. For every $y \in X$, there is a unique $x \in X$ such that $x - A(x) = y$
      ii. There exists some non-zero $x \in X$ such that $x - A(x) = 0$ [4]
   (b) Show that the homogeneous equation $x - A(x) = 0$ has a non-zero solution in X if and only if the transposed homogeneous equation $x' - A'(x') = 0$ has a non-zero solution in X' [4]
   (c) Show that for a given $y \in X$, the equation $x - A(x) = y$ has a unique solution if and only if $x_j'(y) = 0$ for j= 1,2,...,m , where $x_1', x_2', ..x_m'$ form a basis for the solution space of the transposed equation $x' - A'(x') = 0$. Also show that if x_0 is a particular solution of $x - A(x) = y$, the general solution is given by $x = x_0 + k_1x_1 + k_2x_2 + ...k_mx_m$, where $k_1, k_2, ..k_m$ are scalars and $\{x_1, x_2, ...x_m\}$ is a basis for the solution space of $x - A(x) = 0$ [4]

4. (a) Let A_0 be an operator of finite rank on a linear space over K given by $A_0(x) = f_1(x)x_1 + f_2(x)x_2 + ... + f_m(x)x_m, x \in X$ where $x_1, x_2, ..x_m$ are in X and $f_1, f_2, ...f_m$ are linear functionals on X.

   Let $M = \begin{pmatrix} f_1(x_1) & . & . & . & f_1(x_m) \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ f_m(x_1) & . & . & . & f_m(x_m) \end{pmatrix}$.

   If $y_0 \in X$ and $v_0 = (f_1(y_0), ...f_m(y_0))$, then show that $x_0 - A_0(x_0) = y_0$ and $u_0 = (f_1(x_0), ..f_m(x_0))$ if and only if $u_0 - Mu_0 = v_0$and $x_0 = y_0 + u_0(1)x_1 + ... + u_0(m)x_m$ [6]
   (b) Let X be a Banach space and $A \in CL(X)$. For $n = 1,2,...$ let $P_n \in BL(X)$ be a projection maps of finite rank and $A_n^P = P_nA, A_n^S = AP_n, A_n^G = P_nAP_n$. Show that if $P_n(x) \to x$ in X , for every $x \in X$, then $\| A_n^P - A\| \to 0$ .If in addition $P_n'(x') \to x'$ in X' for every $x' \in X'$, then $\|A_n^s - A\| \to 0$ and $\|A_n^G - A\| \to 0$[6]

Unit III

5. (a) Let H be a Hilbert space and $A \in BL(H)$ .Prove that $\|A\| = \|A^*\|$ and $\|A\|^2 = \|A^*A\| = \|AA^*\|$ [4]

   (b) Define normal operator . Show that $A \in BL(H)$ is normal if and only if $\|A^*\| = \|A(x)\| \forall x \in X$. Also show that in this case $\|A\|^2 = \|A^*A\| = \|A^2\|$ [5]

   (c) Let H be Hilbert space and $A \in Bl(H)$ .Show that $\|A\| = sup\{| \langle A(x), x\rangle | : x \in H \|x\| \leqslant 1\}$ Also show that $A = 0$ if and only if $\langle A(x), x\rangle = 0 \, \forall x \in H$ [3]

6. (a) Let H be Separable Hilbert space and $\{u_1, u_2, ..\}$ be an orthonormal basis for H .If $\{k_n\}$ is a bounded sequence of real numbers ,show that the operator given by $A(x) = \sum_n k_n \langle x, u_n\rangle u_n$, $x \in H$ is positive if and only if each k_n is non negative. [4]

   (b) If H is a Hilbert space over $\mathbb{C}$ and $A \in BL(H)$,show that there are unique self-adjoint operators B and C on H such that $A = B + iC$. [4]

   (c) Product of two positive operators is a positive operator .True or False ?,Justify your answer . [4]

Unit IV

7. (a) Let H be a Hilbert space and $A \in BL(H)$. Define numerical range of A. Show that $\sigma(A)$ is contained in the closure of w(A).

   (b) Let A be a self-adjoint operator on a finite dimensional Hilbert space .Show that every root of the characteristic polynomial of A is real. [5]

   (c) Let H be a non-zero Hilbert space and A a self-adjoint operator on H . Show that $\{m_A, M_A\} \subset \sigma_a(A) \subset \sigma(A) \subset [m_A, M_A]$ [3]

8. (a) Define Hilbert Schmidt operator .If A is a Hilbert Schmidt operator on a Hilbert space , Show that A is compact . [4]

   (b) If A is a compact self- adjoint operator on a non -zero Hilbert space ,Show that $\|A\|$ or $-\|A\|$ is an eigenvalue of A . [4]

   (c) Let H be a finite dimensional Hilbert space and $A \in BL(H)$.If $\mathbb{K} = \mathbb{C}$ and A is normal or $\mathbb{K} = \mathbb{R}$ and A is self-adjoint ,Show that there is an orthonormal basis for H consisting of eigenvectors of A. In that case give the expression for A interms of the orthonormal basis elements and eigenvalues. [4]

Unit V

9. (a) Let $S : D \to C([a,b])$ be the Sturm - Liouville operator , given by $S(x) = -x" + q(x)$ .Show that

   i. $\langle S(x), y\rangle = \langle x, S(y)\rangle \, \forall x, y \in D$

   ii. Show that the eigenvalues of S are real and if μ is an eigenvalue of S ,then there is a corresponding real valued eigenfunction u such that $\langle u, u\rangle = 1$ and any other eigenfunction corresponding to μ is a constant multiple of u

   iii. If μ and ν are distinct eigenvalues of S with u and v the corresponding eigenfunctions in D then prove that $\langle u, v\rangle = 0$

   [12]

10. (a) Define Green function for a Sturm - Liouville operator. If 0 is not an eigenvalue of S ,for each fixed $s \in (a,b)$ prove that $\lim_{t \to s^-} \frac{dk_s}{dt} - \lim_{t \to s^+} \frac{dk_s}{dt} = 1$ [7]

    (b) If 0 is not an eigenvalue of the Sturm -Liouville operator and $A : L^2([a,b]) \to L^2([a,b])$ is defined by $A(y)(s) = \int_a^b k(s,t)y(t)dm(t), y \in L^2([a,b]), s \in [a,b]$ where k(,) is the Green function of S then prove that μ is anon-zero eigenvalue of S if and only if $\frac{1}{\mu}$ is an eigenvalue of A [5]

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Fourth Semester M.Sc.(Mathematics)Model Question Paper
(CCSS-2015 Admission)
Kannur University
MAT E 401 : ADVANCED COMPLEX ANALYSIS

Time : 3 Hours Max. Marks : 60

Instructions : *1) Answer **any one** question from **each** Unit.*
*2) **Each** question carries **12** marks.*

UNIT – I

1. a) Prove that $\pi \cot \pi z = \frac{1}{z} + \sum_{n \neq 0} \left(\frac{1}{z-n} + \frac{1}{n} \right)$. 6

   b) Prove that every function which is mesomorphic in the whole plane is the quotient of two entire functions. 6

2. a) Prove that $\sin \pi z$ is an entire function of genus 1. 6

   b) Prove that $\Gamma(Z) = \int_0^\infty e^{-t} z^{t-1} dt$. 6

UNIT – II

3. State and prove the Riemann mapping theorem. 12

4. a) Let f be a topological mapping of a region Ω onto a region Ω'. If $\{z_n\}$ or z(t) tends to the boundary of Ω, then prove that $\{f(z_n)\}$ or f(z(t)) tends to the boundary of Ω'. 5

   b) Define analytic arc and give one example. 2

   c) Explain the behaviour of a conformal mapping at an angle. 5

UNIT – III

5. a) Define mean value property and prove that a continuous function u(z) which satisfies the mean value property is necessarily harmonic. 5

   b) Prove that a continuous function v(z) is subharmonic in Ω if and only if it satisfies the inequality $v(z_0) \leq \frac{1}{2\pi} \int_0^{2\pi} v(z_\theta + re^{i\theta}) d\theta$ for every disk $|z - z_0| \leq r$ contained in Ω. 7

P.T.O.

6. a) Write a short note on the Dirichlet's problem. 3

b) Prove that the function u(z) defined by $u(z) = \text{l.u.b.}\,(v(z))$ for $v \in \mathfrak{B}(f)$ is harmonic in Ω. 9

UNIT – IV

7. a) Define a simply periodic function and obtain the Fourier development. 5

b) Prove that any two bases of the same module are connected by a unimodular transformation. 7

8. a) Define elliptic function and give one example. 3

b) Show that the sum of residues of an elliptic function is zero. 4

c) Define modular function and obtain the functional equations to be satisfied by an elliptic modular function under a modular transformation. 5

UNIT – V

9. a) Define a sheaf over an open set D in the complex plane. Prove that a sheaf $\mathfrak{S}$ over D is the sheaf of germs of analytic functions over the whole complex plane. 7

b) Define analytic continuation along arc. Prove that two analytic continuations $\bar{\gamma}_1$ and $\bar{\gamma}_2$ of a global analytic function f along the same arc γ are either identical or $\bar{\gamma}_1(t) \neq \bar{\gamma}_2(t)$ for all t. 5

10. a) Write a short note on branch points. 3

b) Prove that the fundamental group of the punctured disk is isomorphic to the additive group of integers. 5

c) Distinguish between Section and Riemann surface. 4

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Fourth Semester M.Sc.(Mathematics)Model Question Paper
(CCSS-2015 Admission)
Kannur University
MAT E 402 : GRAPH THEORY

Time : 3 Hours Max. Marks : 60

Instructions : *Answer **one** question from **each** Unit. **Each** question carries **12** marks.*

UNIT – I

1. a) Prove that in any group of n persons ($n \geq 2$), there are atleast two with the same number of friends. 4

   b) If G is simple and has n vertices, then prove that G is connected if $\delta \geq \frac{n-1}{2}$. 4

   c) Show that if G is a self complementary graph of order n, then $n \equiv 0$ or 1 (mod 4). 4

2. a) Prove that every tournament contains a directed Hamilton path. 8

   b) Explain the following concepts with **one** example **each** :

      i) k – partite tournament ($k \geq 2$)

      ii) Score vector of a tournament. 4

UNIT – II

3. a) Prove that a simple cubic graph G, which is connected, has a cut vertex if and only if it has a cut edge. 6

   b) State and prove the edge version of Menger's theorem for undirected graphs. 6

4. a) Prove that every connected graph contains a spanning tree. 4

   b) Prove that for a simple connected graph G, its line graph L(G) is isomorphic to G if and only if G is a cycle. 4

   c) Define :

      i) Centre

      ii) Centroid of a tree. Give an example of a tree with disjoint centre and centroid. 4

P.T.O.

UNIT – III

5. a) Define the following :
   i) $\alpha(G)$
   ii) $\beta(G)$
   iii) $\alpha'(G)$
   iv) $\beta'(G)$. 2

   b) For any graph G, prove that $\alpha + \beta = n$, where n is the order of G. 4

   c) Prove that a matching M of a graph is maximum if and only if G has no M-augmenting path. 6

6. a) State and prove Ore's theorem. 6

   b) Define closure of a graph G. Prove that closure of a graph G is well defined. 6

UNIT – IV

7. a) Prove that for any simple graph G of order n, prove that $2\sqrt{n} \le \chi + \chi^c \le n+1$ and $n \le \chi\chi^c \le \left(\frac{n+1}{2}\right)^2$. 6

   b) If G is a loopless bipartite graph, prove that $\chi'(G) = \Delta(G)$. 6

8. a) Define a k- critical graph. Prove that if a simple graph G is k-critical, then $\delta \ge k-1$. Also prove that $\chi \le 1 + \Delta$. 6

   b) Prove that a simple graph G on n vertices is a tree if and only if $f(G, \lambda) = \lambda(\lambda - 1)^{n-1}$. 6

UNIT – V

9. a) Define planar graph. Give one example. Show that the complement of a simple planar graph with ll vertice's is non planar. 4

   b) Prove that every planar graph is 6-vertex colourable. 4

   c) Prove that graph is planar if and only if it is embeddable on a sphere. 4

10. a) Prove that a graph G is planar if and only if each of its blocks is planar. 6

    b) Prove that K_5 is non planar. 4

    c) Define maximal planar graph. Give an example. 2

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Fourth Semester M.Sc.(Mathematics) Model Question Paper
(CCSS-2015 Admission)
Kannur University

MAT E 403 : Coding Theory

Time : 3 Hours

Max. Marks : 60

Instructions : *1) Answer **any one** question from **each** Unit.*
*2) **Each** question carries **12** marks, students have to answer **one** question from **two** questions given from **each** Unit.*

UNIT – 1

1. a) Show that a code of distance d will correct all error pattern of weight $\leq [(d-1)/2]$ and there is at least one error pattern of weight $1 + [(d-1)/2]$. 7

b) Let C = {001, 101, 110}. Does C corrects the error patterns u = 100, v = 000. 5

2. a) Suppose we have a BSC with $1/2 < p < 1$. Let v_1 and v_2 be code words and w a word, each of length n. Suppose that v_1 and w disagree in d_1 position and v_2 and w disagree in d_2 position. Then show that $\varphi_p(v_1, w) \leq \varphi_p(v_2, w)$ iff $d_1 \geq d_2$. 4

b) Suppose |M| = 2, n = 3, C = {000, 111}. If v = 000 is sent, when will IMLD conclude this correctly and when will IMLD incorrectly conclude that 111 was sent ? 2

c) Construct the IMLD table for the code C = {0000, 1010, 0111}. 4

d) p = .90, |M| = 3, n = 4 and C = {0000, 1010, 0111}. For v = 0000 in C, calculate θp (C, v). 2

UNIT – 2

3. a) Prove that equivalent linear codes always have the same length, dimension and distance. 6

b) Find basis for the linear code $C = \langle S \rangle$ and $C^{\perp}$ for the set S = {11101, 10110, 01011, 11010}. 6

P.T.O.

4. a) Explain the following with suitable examples :
   i) Linear code
   ii) Parity check matrix
   iii) Systematic Code. 7

   b) Calculate θp (C) for the code C = {0000, 1001, 0101, 1100}. 5

UNIT – 3

5. a) Define perfect code. 1
   b) Show that Hamming Codes are perfect. 3
   c) State and prove Hamming bound theorem. 4
   d) What is a lower and an upper bound on the size or dimension k of a linear code with n = 9, d'= 5 ? 4

6. a) Describe the extended Golay Code. 3
   b) Show that the extended Golay Code has the distance 8. 6
   c) Decode W = 001001001101, 101000101000. 3

UNIT – 4

7. a) Let $a \leftrightarrow a(x)$, $b \leftrightarrow b(x)$ and $b^1 \leftrightarrow b^1(x) = x^n b(x^{-1}) \bmod (1 + x^n)$. Then show that $a(x)\, b(x) \pmod{1 + x^n} = 0$ if and only if $\pi^k(a).\, b^1 = 0$ for $k = 0, 1, 2, 3 .., n$. 3
   b) If C is a linear code of length n and dimension k with generator g (x) and if $1 + x^n = g(x)\, h(x)$, show that $C^{\perp}$ is a cyclic code of dimension n – k with generator $x^k h(x^{-1})$. 6
   c) Find the number of proper linear cyclic code of length 56. 3

8. a) If $f(x) \equiv g(x) \pmod{h(x)}$, then show that $f(x) + p(x) \equiv (g(x) + p(x)) \bmod h(x)$. 3
   b) Compute f (x) (mod h (x)) and its corresponding word for $f(x) = 1 + x + x^2$, $h(x) = 1 + x^3 + x^5$. 3
   c) Find a basis for the smallest linear cyclic of length 7 containing the word v = 1101000. 6

UNIT – 5

9. a) Let $\alpha \neq 0$ be an element of GF (2^r). Let $m_\alpha(x)$ be the minimal polynomial of α. Then prove the following : 6

   i) $m_\alpha(x)$ is irreducible over K.

   ii) If f (x) is any polynomial over K such that $f(\alpha) = 0$, then $m_\alpha(x)$ is a factor of f (x).

   iii) The minimal polynomial is unique.

   b) Find the minimal polynomial of $\alpha = \beta^3$, $\alpha \in$ GF (2^4) constructed using $h(x) = 1 + x + x^4$. 6

10. a) Construct GF (2^2) using $1 + x + x^2$. 4

   b) Let C be the RS (4, 2) with $g(x) = \beta + x$ where GF (2^2) is constructed using $1 + x + x^2$.

   Then :

   i) Find n, k, d and |C| for this code.

   ii) Construct a generating matrix G for C.

   iii) Find all the code words in C, their corresponding binary code words in $\hat{C}$ and the corresponding messages. 8

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Fourth Semester M.Sc(Mathematics) Model Question Paper
(CCSS-2015 Admission)
Kannur University
MAT E 404 : NUMBER THEORY

Time : 3 Hours Max. Marks : 60

Instructions : *Answer **one** question from **each** Unit.*
__Each__ question carries __12__ marks.

UNIT – I

1. a) State and prove Euclid's division lemma. **4**

   b) Prove that if a and b are integers, not both zero, then gcd (a, b) exists and is unique. **5**

   c) If d is the gcd of 299 and 481, find integers x and y such that $299x + 481y = d$. **3**

2. a) State and prove fundamental theorem of arithmetic. **5**

   b) State and prove Format's little theorem. **4**

   c) Prove that $-\log(1-x)$ is the generating function for the sequence $a_1 = 1$, $a_2 = -1/2 \ldots a_n = 1/n, \ldots$ **3**

UNIT – II

3. a) State and prove Möbius Inversion formula. **4**

   b) Prove that if $\sigma_k(n) = \sum_{d/n} d^k$, then $\sigma_k(n)$ is multiplication. **4**

   c) Explain the concept of primitive roots with suitable examples. **4**

4. a) Prove that for each prime p, there exists primitive roots modulo p. **6**

   b) State and prove Tchebychev's theorem. **6**

P.T.O.

UNIT – III

5. a) Prove that the number a is a quadratic residue modulo p if and only if $a^{\frac{p-1}{2}} \equiv 1 \pmod{p}$. **4**

b) State and prove Gauss's Lemma. **4**

c) Use Gauss's lemma to show that 17 is a quadratic residue modulo 19. **4**

6. a) If p is an odd prime and gcd (a, p) = 1, then show that the congruence $x^2 \equiv a \pmod{p^n}$ has a solution iff $\left(\frac{a}{p}\right) = 1$. **6**

b) Use quadratic reciprocity law to prove that $\left(\frac{3}{p}\right) = \begin{cases} 1 & \text{if } p \equiv 1 \text{ or } 11 \pmod{12} \\ -1 & \text{if } p \equiv 5 \text{ or } 7 \pmod{12} \end{cases}$. **6**

UNIT – IV

7. Prove that a positive integer n can be represented as a sum of two squares if and only if its factorization into powers of distinct primes contains no odd powers of primes congruent to 3 modulo 4. **12**

8. Prove that every positive integer is a sum of four non negative integral squares. **12**

UNIT – V

9. a) If $\lim\limits_{n \to \alpha} a_n = L$, then $\lim\limits_{x \to l^-} (1 - x) \sum\limits_{n=0}^{\infty} a_n x^n = L$. **6**

b) Prove that $\lim\limits_{N \to \infty} \frac{1}{N} \sum\limits_{n=0}^{N} r_2(n) = \pi$. **6**

10. State and prove Dirichlet's divisor problem. **12**