KANNUR UNIVERSITY

(Abstract)

M.Sc. Mathematics Programme under Choice Based Credit Semester System in the University Department– Revised Scheme, Syllabus & Model Question Papers Implemented with effect from 2016 admission- Orders issued.

ACADEMIC 'C'SECTION

U.O. No.Acad/C4/ 6206/2015 Civil Station P.O, Dated,10 -08-2016

Read: 1. U.O No.Acad/C3/2049/2009 dated 11.10.2010.
2. U.O No.Acad/C3/2049/2009 dated 05.04.2011.
3..U.O No.Acad/C4/14536/2014 dated 29.05.2015.
4. Meeting of the syndicate sub – committee held on 16/01/2015.
5. U.O.No.Acad/C4/6206/2015 dated 30.10.2015
6. Meeting of the Department Council held on 18.06.2016.
7. Letter from the HOD,No. Maths/517/2016 dt:15/07/2016,Dept.of Mathematical Sciences, Mangattuparamba campus.

ORDER

1.The Regulations for Post Graduate Programmes under Choice Based Credit Semester System were implemented in the Schools/Departments of the University with effect from 2010 admission as per the paper read (1) above and certain modifications were effected to the same vide paper read (2) and (3).

2. The meeting of the Syndicate Sub-Committee recommended to revise the Scheme and Syllabus of all the Post Graduate Programmes in the University Schools/Departments under Choice Based Credit Semester System (CCSS) with effect from 2015 admission vide paper read (4) above and the Scheme, Syllabus & Model Question papers for M.Sc Mathematics programmes was implemented in the university department w.e.f 2015 admission vide paper read (5) above.

3.The Department Council vide paper read (6) above recommended to revise the scheme & syllabi of M.Sc Mathematics Programme w.e.f 2016 admission and approved the Scheme, Syllabus & Model Question Papers for M.Sc. Mathematics Programme under Choice Based Credit Semester System(CCSS) for implementation with effect from 2016 admission

4.The HOD, Dept. of Mathematical Sciences vide paper read (7) above, has forwarded the Scheme, Syllabus & Model Question Papers for M.Sc Mathematics Programme under Choice Based Credit Semester System for implementation with effect from 2016 admission.

P.T.O

5. The Vice Chancellor after considering the matter in detail, and in exercise of the powers of the Academic Council conferred under section 11(1) of KU Act 1996, and all other enabling provisions read together with, has accorded sanction to implement the Scheme, Syllabus & Model Question Papers for M.Sc. Mathematics Programme under Choice Based Credit Semester System, offered in the University Department w.e.f 2016 admission, subject to report to the Academic Council.

6. Orders are, therefore, issued accordingly.

7. The revised Scheme, Syllabus and Model Question Papers effective from 2016 admission are appended.

Sd/-
JOINT REGISTRAR (ACADEMIC)
FOR REGISTRAR

To

The HOD, Department of Mathematical Sciences
Mangattuparamba Campus, Kannur University

Copy To:

1. The Examination Branch (through PA to CE)
2. PS to VC/PA to PVC/PA to R/PA to CE/PA to FO
3. JR/AR I Academic
4. The Computer Programmer *(for uploading in the website)*
5. SF/DF/FC

Forwarded/By Order

SECTION OFFICER

KANNUR UNIVERSITY CIVIL STATION P.O. KANNUR Pin-670 002 THAVAKKARA

For more details: log on www.kannur university .ac.in

KANNUR UNIVERSITY
DEPARTMENT OF MATHEMATICAL SCIENCES

Regulations, Scheme and Syllabus for
M.Sc. Mathematics Programme
With Effect From 2016 Admission

KANNUR UNIVERSITY
DEPARTMENT OF MATHEMATICAL SCIENCES

Regulations, Scheme and Syllabus for M.Sc. Mathematics Programme With Effect From 2016 Admission

1. Eligibility for Admission

The essential qualification for admission shall be a B.Sc. degree in Mathematics with at least 55% marks or equivalent CGPA in the core and complimentary(main and subsidiary) subjects together.

2. Mode of Selection

The selection of the candidates will be on the basis of the marks secured in the entrance test.

3.Mode of Instruction : English

4. Duration of the programme

The duration of the M.Sc.Programme shall be 2 years, each year comprising two semesters. The duration of each semester shall be 6 months inclusive of examinations.

5. Course Structure

The programme is offered under **CCSS** with duration 2 years (4 semesters). Credit defines the quantum of contents/syllabus prescribed for a course and determines the number of hours of instruction required per week. There shall be at least sixteen week schedule per semester to complete the course contents.

The department council will assign an advisor(faculty member) to each of the student admitted. He/She advice the student about the academic programme and counsel on the choice of the course depending on the student's academic background and objective.

The department council shall prescribe the maximum number of students that can be admitted, taking into consideration of the facilities available. The minimum duration for the completion of the M.Sc. Mathematics programme is four semesters and the maximum period for the completion is eight semesters.

No student shall register for more than 24 credits and less than 16 credits per semester.

The minimum total credits required for the successful completion of the M.Sc. programme is 80 and in which minimum credit required for the core course is 60 and the minimum for the elective is 12.

Those who secure only the minimum credit for core/elective subject has to supplement the deficiency required for obtaining the minimum total credits required for the successful completion of the programme from the other division

6. Evaluation

All the semesters will have continuous and end semester assessments. The course instructors carry out the internal evaluation for each paper. For theory papers, the proportion of the distribution of marks among the continuous evaluation and end semester examination shall be 40:60. Duration of the theory examination is 3 hours.

6.1 Continuous Evaluation

(i) Theory paper

Continuous evaluation includes assignments, seminars, periodic written examinations and end semester viva-voce for each paper.
Weightage to the components of the continuous assessment shall be

Written test papers	:	40%	(16 marks)
Assignments	:	20%	(8 marks)
Seminar /Viva	:	40%	(16 marks)

(ii) Viva - Voce

a) Internal evaluation : (Maximum marks 20): Marks for Viva – Voce will be awarded based on viva conducted by internally.

b) External evaluation: (Maximum marks 30) :A comprehensive end semester external Viva - Voce shall be conducted by two examiners one external and one internal. External examiner for Viva - Voce shall be selected from the approved list of experts provided by the Head of the Department / Chairman BOE.

6.2 End Semester Evaluation

i) The end semester evaluation of each theory paper consist of a written examination (Maximum marks 60, 3 Hours duration)

For written examination, two questions shall be asked from each unit in the syllabus, of which the student is expected to answer any one and the marks for each question shall be 15

ii) A comprehensive external Viva - Voce shall be conducted by two examiners one external and one internal. External examiner for Viva - Voce shall be selected from the approved list of experts provided by the Head of the Department / Chairman BOE.

6.3 Project work

Each M.Sc. students has to carry out a research project during third and fourth semesters. The project work should be started in the third semester and should go continuously for the third and fourth semesters

The project evaluation, comprising of internal (total 80 marks) and external (total 120 marks), will be carried out during fourth semester. The scheme of evaluation of project is as follows.

Total marks : 200

Content	:	30%	60 (36 external & 24 internal)
Methodology and Presentation	:	50%	100 (60 external & 40 internal)
Dissertation Viva-voce	:	20%	40(24 external & 16 internal)

6.4 Re-appearance of the end semester examination

A minimum grade point 5(Grade C) is needed for the successful completion of a course. A student who has failed in a course can reappear for the end semester examination of the same course along with the next batch or choose another course in the subsequent semesters to acquire the minimum credits needed for the completion of the programme. A student who fails in any paper need to appear for re-examination in that paper only. There shall be no supplementary examination, normally no student shall allowed taking more than eight consecutive semesters for completing the programme from the date of enrollment.

SYLLABUS

The syllabus appended is applicable from 2016 admission.

M.Sc. Mathematics under CCSS

(2016 Admission)

Course Structure

Course Code	Course Title	Number of hours per week		Credits	CE	ESE	Total
		Lecture hours	Tutorial hours				
First Semester							
MAT C 101	Algebra I	4	2	3	40	60	100
MAT C 102	Linear Algebra	4	2	3	40	60	100
MAT C 103	Differential Equations I	4	2	3	40	60	100
MAT C 104	Real Analysis	4	2	3	40	60	100
MAT C 105	Topology	4	2	3	40	60	100
MAT C 106	Viva-voce	-	-	3	20	30	50
Total		**20**	**10**	**18**	**220**	**330**	**550**
Second Semester							
MAT C 201	Complex Analysis	4	2	4	40	60	100
MAT C 202	Functional Analysis I	4	2	4	40	60	100
MAT C 203	Algebra II	4	2	4	40	60	100
MAT C 204	Differential Equations II	4	2	4	40	60	100
MAT C 205	Measure & Integration	4	2	4	40	60	100
MAT C 206	Viva-voce	-	-	2	20	30	50
Total		**20**	**10**	**22**	**220**	**330**	**550**
Third Semester							
MAT C 301	Differential Geometry	4	2	4	40	60	100
MAT C 302	Functional Analysis II	4	2	4	40	60	100
MAT C 303	Probability	4	2	4	40	60	100
MAT E 304	Elective I	4	2	4	40	60	100
MAT C 305	Dissertation	-	6	-	-	-	-
MAT C 306	Viva-voce	-	-	2	20	30	50
Total		**16**	**14**	**18**	**180**	**270**	**450**

Fourth Semester							
MAT E 401	Elective 2	4	2	3	40	60	100
MAT E 402	Elective 3	4	2	3	40	60	100
MAT E 403	Elective 4	4	2	3	40	60	100
MAT E 404	Elective 5	4	2	3	40	60	100
MAT C 405	Dissertation	-	6	8	80	120	200
MAT C 406	Viva-voce	-	-	2	20	30	50
Total		**16**	**14**	**22**	**260**	**390**	**650**
Total							**2200**

List of Electives: (To be added/chosen as per the requirements and availability of faculty)

1. Algebraic Geometry
2. Projective Geometry
3. Advanced Complex Analysis
4. Analytical Mechanics
5. Stochastic Processes
6. Fluid Mechanics
7. Algebraic Topology
8. Numerical Analysis and Computing
9. Graph Theory
10. Fractal Geometry
11. Coding Theory
12. Cryptography
13. Number Theory
14. Analytic Number Theory
15. Algebraic Number Theory
16. Fuzzy Mathematics
17. Operations Research

DETAILED SYLLABUS

MAT C 101 ALGEBRA I

Unit 1

Direct products and finitely generated abelian groups. Homomorphisms a,Factor groups. Factor group computations and simple groups. (Chapter 2 Section11 and Chapter 3 Sections 13-15 of Text .)

Unit 2

Group Action on a set,Application of G-sets to counting,Sylow theorems,Applications of the Sylow theory. Free abelian groups. (Chapter 3 Section16,17 and Chapter 7 Sections 36,37,38 of Text)

Unit 3

Free groups. Group presentation. The Field of quotients of an integral domain. Ring of polynomials. (Chapter 7 Sections 39-40, Chapter 4 Sections 21,22 of Text .)

Unit 4

Factorisation of polynomials over a field. Homomorphisms and factor rings. Prime and maximal ideals. (Chapter 4 Section 23; Chapter 5Sections 26,27 of Text .)

Text Books:

1. J. B. Fraleigh – A First Course in Abstract Algebra- Narosa (7th edn., 2003)

Reference:
1. I.N. Herstein – Topics in Algebra- Wiley Eastern
2. J.A.Gallian – Contemporary Abstract Algebra
3. Hoffman & Kunze – Linear Algebra – Prentice Hall
4. M. Artin, Algebra, Prentice Hall, 1991

MAT C 102: LINEAR ALGEBRA

Unit 1

Linear Transformations: Linear Transformations, The Algebra of Linear Transformations, Isomorphism, Representation of Transformation by Matrices, (Chapter-3; Sections 3.1, 3.2,3.3, 3.4,)

Unit 2

Linear Functionals, The Double Dual, The Transpose of a Linear Transformation.
Elementary Canonical Forms: Introductions, Characteristic Values
(Chapter 3, sections 3.5, 3.6, 3.7 Chapter-6: Section 6.1, 6.2,)

Unit 3

Annihilating Polynomials ,Invariant Subspace, Simultaneous Triangulations& Simultaneous Diagonalisation.

Elementary Canonical Forms: Invariant Direct Sums,
(Chapter-6: Sections 6.3, 6.4, 6.5, 6.6 6.7)

Unit 4

The Primary Decomposition Theorem.
The Rational and Jordan Forms: Cyclic Subspaces and Annihilators, Cyclic Decomposition and the Rational Forms

Inner Product Spaces: Inner Products, Inner Product Spaces, (Chapter 6 section 6.8; Chapter-7: Sections: 7.1, 7.2, Chapter-8: Sections 8.1, 8.2,)

Text Book: Kenneth Hoffman & Ray Kunze; Linear Algebra; Second Edition, Prentice-Hall of India Pvt. Ltd

Reference:

1. Stephen H. Friedberg, Arnold J Insel and Lawrence E. Spence: Linear Algebra: 4th Edition 2002: Prentice Hall.
2. Serge A Land: Linear Algebra; Springer
3. Paul R Halmos Finite-Dimensional Vector Space; Springer 1974.
4. McLane & Garrell Birkhoff; Algebra; American Mathematical Society 1999.
5. Thomas W. Hungerford: Algebra; Springer 1980
6. Neal H.McCoy & Thomas R.Berger: Algebra-Groups, Rings & Other Topics: Allyn & Bacon.
7. S Kumaresan; Linear Algebra A Geometric Approach; Prentice-Hall of India 2003.

MAT C 103 DIFFERENTIAL EQUATIONS I

Unit 1

Existence and Uniqueness of solutions of differential equations. Oscillation theory (Chapter 13 sections 68 , 69and 70, Chapter 4 complete)

Unit 2

Power series solutions and special functions.(excluding section 26), Series solutions of first order equations, Second order linear equations, Regular singular points,Gauss's hyper geometric equation, point at infinity (Chapter 5, Section 27-32)

Unit 3

Legendre polynomials and their properties , Bessel function and their properties. Application of Legendre polynomial to potential theory, Systems of first order equations: linear systems ,homogeneous linear system with constant coefficients and nonlinear systems (Chapter 8, Sections 44-47, Appendix A and Chapter 10 sections 54-56)

Unit 4

Nonlinear equations: Autonomous systems, Th phase plane and its phenomina, Types of critical points , stability, Critical points and stability for linear systems, Stability by Liapunov's direct method, simple critical points of nonlinear systems (Chapter 11, Sections 58-62)

Text: George F. Simmons – Differential Equations with applications and historical notes. Tata McGraw Hill, 2003

References:

1. Birkhoff G &G.C. Rota – Ordinary Differential Equations – Wiley
2. E.A. Coddington – An introduction to Ordinary Differential Equations – Prentice Hall India
3. Chakrabarti – Elements of Ordinary Duifferential Equations & Special Functions – Wiley Eastern

MAT C 104 REAL ANALYSIS

Unit 1

Basic Topology-Finite,Countable and uncountable Sets Metric spaces, Compact Sets ,Perfect Sets, Connected Sets.

Continuity-Limits of function, Continuous functions, Continuity and compactness, continuity and connectedness, Discontinuities, Monotonic functions, Infinite limits and Limits at infinity.

Unit 2

Differentiation, Derivative of a real function. Mean value theorems, Continuity of derivatives. L Hospital's rule. Derivatives of higher order. Taylor's theorem. Differentiation of vector valued functions

Unit 3

Reimann – Stieltjes integral. Definition and existence of the integral. Integration and differentiation. Integration of vector – valued functions. Rectifiable curves.

Unit 4

Sequences and series of functions. Uniform convergence. Uniform convergence and continuity. Uniform convergence and differentiation.

Equicontinuous families of functions. Stone – Weierstrass theorem

.

Text: Walter Rudin – Principles of Mathematical Analysis (3rd edition) – Mc Graw Hill, Chapters2,4, 5,6, and 7(up to and including 7.27 only)

References:

1. T.M. Apostol – Mathematical Analysis (2nd edition) – Narosa
2. B.G. Bartle – The Elements of Real Analysis – Wiley International
3. G.F. Simmons – Introduction to Topology and Modern Analysis – McGraw Hill

MAT C 105 TOPOLOGY

Unit 1

Naïve set theory. Topological spaces. Connected compact spaces.

Unit 2

Continuous functions. Product spaces. The Tychnoff theorem

Unit 3

Separation axioms. Separation by continuous functions. More separability.

Unit 4

Complete metric spaces. Applications.

Text (for units 1 to 4):

M. Singer and J.A. Thorpe – Lecture Notes on Elementary Topology and Geometry, Springer Verlag 1967 (Chapter 1 and 2)

References:

1. K.D. Joshi – Introduction to General Topology, Wiley Eastern (1983)
2. G.F. Simmons – Introduction to Topology & Modern Analysis – McGraw Hill
3. J.R. Munkres – Topology, a First Course, Prentice Hall India
4. Kelley J.L. – General Topology, von Nostrand

MAT C 201 COMPLEX ANALYSIS

Unit 1

Conformality - Arcs and closed curves, analytic functions in regions, conformal mapping , length and area - Linear transformations- the linear group, the cross ratio, symmetry, oriented circles, families of circles, Elementary conformal mapping-use of level curves, a survey of elementary mappings and Riemann surfaces (Chapter 3, Sections 2,3,4)

Unit 2

Complex integration- Fundemental theorems, lineintegals, rectifiable arcs, Cauchy's theorem for rectangle and disc, Cauchy's integral formula- the index of a point with respect to a closed curve, the integral formula, higher derivatives ,Local properties of analytic functions - removable singularities, Taylor's theorem Zeroes and poles local mapping, , the maximum principle (Chapter 4, Sections 1,2,3)

Unit 3

The general form of cauchy's theorem, Chains and cycles, simple connectivity, homology, general form of Cauchy's theorem, proof of Cauchy's theorem, Locally exact differentials, multiply connected regions , The calculus of residues – the residue theorem, the argument principle , evaluation of definite integrals (Chapter 4, Section 4,5)

Unit 4

Harmonic functions- definition and basic properties, the mean value property, Poisson formula, Schwarz theorem, the reflection principle, Power series expansions -Weierstrass theorem, the Taylor series and the Laurent series (Chapter 4, Sections 6 and Chapter 5 section 1)

Text: L.V.Ahlfors – Complex Analysis (3rd edition) – McGraw Hill International (1979)

References:

1.Conway J.B. – Functions of One Complex Variable – Narosa

2. E.T.Copson – An Introduction to the Theory of Complex Variables - Oxford

MAT C 202 FUNCTIONAL ANALYSIS I

UNIT 1

Metric spaces and Continuous Functions (section 3, 3.1 to 3.3 & 3.4(without proof), 3.11 to 3.13) Lp spaces , Fourier series and Integrals (section 4.5 to 4.11),

UNIT 2

Normed spaces(section 5).Continuity of linear maps (section 6), Hahn-Banach Theorems (section 7, omit Banach limits),

UNIT 3

Banach spaces (section 8),Uniform Boundedness Principle (section 9, omit Quadrature Formulae and Matrix Transformations and Summability Methods).

UNIT 4

Inner product spaces , Orthonormal sets (Sections 21 and 22), Approximation and Optimization(section 23 , except 23.6)

Text: B.V. Limaye – Functional Analysis (2nd edition) – New Age International, 1996.

REFERENCES

1. E. Kreyszig, Introductory Functional Analysis with Applications (Addison-Wesley)
2. B. Bollabas, Linear Analysis, Cambridge University Press (Indian Edition), 1999.
3. G. F. Simmons, Introduction to Topology and Modern Analysis, McGraw Hill, 1963.
4. A. E. Taylor, D. C. Lay, Introduction to Functional Analysis 2nd edition, Wiley New York, 1980.
5. M.T. Nair, Functional Analysis: A First Course, Wiley Eastern, 1981
6. R. Bhatia. : Notes on Functional Analysis TRIM series, Hindustan Book Agency
7. Kesavan S, : Functional Analysis TRIM series, Hindustan Book Agency
8. S David Promislow : A First Course in Functional Analysis Wiley Interscience, John wiley & Sons, INC., (2008).
9. Sunder V.S, : Functional Analysis TRIM Series, Hindustan Book Agency
10. George Bachman & : Functional Analysis Lawrence Narici Academic Press, NY (1970)
11. Kolmogorov and Fomin S.V. : Elements of the Theory of Functions and Functional Analysis. English Translation, Graylock Press Rochaster NY (1972)
12. W. Dunford and J. Schwartz : Linear Operators Part 1, General Theory John Wiley & Sons (1958)
13. E.Kreyszig : Introductory Functional Analysis with Applications John Wiley & Sons (1978)
14. F. Riesz and B. Nagy : Functional Analysis Frederick Unger NY (1955)
15. J.B.Conway : Functional Analysis Narosa Pub House New Delhi (1978)
16. Walter Rudin : Functional Analysis TMH edition (1978)
17. Walter Rudin : Introduction to Real and Complex Analysis TMH edition (1975)
18. J.Dieudonne : Foundations of Modern Analysis Academic Press (1969)

19. Yuli Eidelman, Vitali Milman : Functional analysis An Introduction,and Antonis Tsolomitis Graduate Studies in Mathematics Vol. 66 American Mathematical Society 2004.

MAT C 203 ALGEBRA II

Unit 1

Unique factorization domains, Euclidean domains; Gaussian integers and multiplicative norms. (Chapter 9 Sections 45,46,47 of Text 1.)

Unit 2

Introduction to extension fields. Algebraic extensions.Geometric constructions. (Chapter 6 Sections 29, 31, 32.)

Unit 3

Finite fields. Automorphisms of fields ,The isomorphism extension theorem,. splitting fields (Chapter 6 Section33, Chapter 10 Sections 48,49,50 of Text 1.)

Unit 4

splitting fields, separable extensions,Totally inseparable extensions , Galois theory,Illustrations of Galois theory (Chapter 10 Section 51,52,53,54)

Text Books:

1. Fraleigh – A First Course in Abstract Algebra- Narosa (7th edn.), 2003

Reference:

1. J.A.Gallian – Contemporary Abstract Algebra
2. Hoffman & Kunze – Linear Algebra – Prentice Hall
3. P.B. Bhattacharya, S.K. Jain, S.R. Nagpal – Basic Abstract Algebra
4. M. Artin – Algebra, Prentice Hall, 1991

MAT C 204 DIFFERENTIAL EQUATIONS II

Unit 1

First order partial differential equations (PDE): Curves and surfaces,Genisis of first order PDE, classification of integrals, linear equations, Pfaffian equations, compatible systems, Charpit's Method, Jacobi's method. (Chapter 1 Sections 1.1 to1.8)

Unit 2

Integral surfaces through a given curve, Quasilinear equations, nonlinear equations Genesis and classification of second order PDE (Chapter 1 Sections 1.9,1.10,1.11 and sections 2.1,2.2 of chapter 2)

Unit 3

Second order equations: classification, one-dimensional wave equation and Laplace's equation (Sections 2.3 and 2.4 of Chapter 2)

Unit 4

Heat conduction problem, Duhamel's principle, families of equipotential surfaces, Kelvin's inversion theorem (Chapter 2 Sections 2.5,2.6,2.8,2.9)

Text : Amarnath – An elementary course in partial differential equations (2nd edition) – Narosa Publishing House, 2003

References: 1. Ian Sneddon – Elements of partial differential equations, McGraw Hill, 1983
2. Phoolan Prasad and Renuka Ravindran – Partial differential equations, New Age

MAT C 205 MEASURE AND INTEGRATION

Unit 1

Introduction. Measurable functions. Measures.

Unit 2

The integral. Integrable functions. Lp – spaces.

Unit 3

Modes of convergence.

Unit 4

Generation of measures. Decomposition of measures.

Text: R.G. Bartle – The Elements of Integration (1966), John Wiley & Sons (Complete Book)

References: 1. H.L. Royden – Real Analysis –Macmillan
2. de Barra – Measure and Integration
3. Inder K. Rana – Measure and Integration – Narosa

MAT C 301 DIFFERENTIAL GEOMETRY

Unit 1

Level sets, vector fields, tangent spaces, surfaces, orientation, Gauss map.

Unit 2

Geodesics, parallel transport, Weingarten map, curvature of plane curves

. **Unit 3**

Arc length, line integrals, curvature of surfaces.

Unit 4

Parametrized surfaces. Local equivalence of surfaces and parametrized surfaces.

Text Book: T. A. Thorpe – Elementary Topics in Differential Geometry, Springer-Verlag, Chs.1-12, 14 and 15.

References:

1. Guillemine & Pollack – Differential Geometry, Prentice Hall
2.Struik D.J. – Classical Differential Geometry – Dover (2nd edn.) (1988)
3.Kreyszig, E. – Introduction to Differential Geometry and Riemannian Geometry – Univ. of Toronto Press (1969)
4. M. Spivak – A Comprehensive Introduction to Differential Geometry Vols. 1-3, Publish or Perish Boston (3rd edn.) (1999)

MAT C 302 FUNCTIONAL ANALYSIS II

TEXT : **LIMAYE , B.V** : FUNCTIONAL ANALYSIS,(2nd Edn.) New Age International Ltd, Publishers New Delhi, Bangalore (1996)

UNIT 1

Closed Graph and Open Mapping Theorems (section 10) , Bounded Inverse Theorems (section 11) , Spectrum of a Bounded Operator (section 12),

UNIT 2

Duals and Transposes (section 13,upto and including 13.6)., Reflexivity (section 16, Omit 16.3 and the proof of 16.5 and 16.6) Definition of Compact
Linear Map, Projection and Riesz Representation Theorems (section 24.1 To 24.6).

UNIT 3

Weak convergence and weak boundedness (section 24.7,24.8)., Bounded Operators and Adjoints (section 25)

UNIT 4

Normal, Unitary and Self Adjoint Operators (section 26, omit Fourier-Plancherel Transform), Spectrum and Numerical Range (section 27), Compact self Adjoint Operators (section 28 , omit 28.7 and 28.8(b)).

REFERENCES:

1. R. Bhatia. : Notes on Functional Analysis TRIM series, Hindustan Book Agency
2. Kesavan S. : Functional Analysis TRIM series, Hindustan Book Agency
3. S David Promislow : A First Course in Functional Analysis Wiley Interscience, John wiley & Sons, INC., (2008.)
4. Sunder V.S. : Functional Analysis TRIM Series, Hindustan Book Agency
5. George Bachman & : Functional Analysis Lawrence Narici Academic Press, NY (1970)
6. Kolmogorov and Fomin S.V. : Elements of the Theory of Functions and Functional Analysis. English Translation, Graylock Press Rochaster NY (1972)
7. W. Dunford and J. Schwartz : Linear Operators Part 1, General Theory John Wiley & Sons (1958)

MAT C 303 PROBABILITY

Unit 1

Probability spaces – Dynkin's theorem, construction of probability spaces, measure constructions. (sections 2.1 to 2.6)

Unit 2

Random variables, elements, and measurable maps – inverse maps, measurable maps, induced probability measures, measurability and continuity, measurability and limits, fields generated by maps.(Sections 3.1, 3.2 except 3.2.2, 3.3 and 3.4) Independence – records, ranks, Renyi theorem, groupings, zero-one laws, Borel-Contelli lemma (Sections 4.1 to 4.6)

Unit 3

Integration and expectation – limits and integrals, infinite integrals the transportation theorem and densities, product spaces, independence and Fubini theorem, probability measures on product spaces. (Sections 5.1 to 5.10 except 5.6)

Unit 4

Convergence concepts – almost sure, convergence in probability, quantile estimation, Lp convergence(sections 6.1 to 6.6 except 6.2.1 and 6.4)

Text: Sidney I Resnick – A Probability Path, Birkhauser (1999) (Chapters 2 to 7)

References:

1. K.L. Chung – Elementary Probability Theory, Narosa
2. W. Feller – Introduction to Probability Theory and Applications volumes & II, John Wiley, 1968
3. A. K. Basu, Measure and Probability, PHI (2004)

<u>ELECTIVES</u>

1. ALGEBRAIC GEOMETRY

Unit 1

Affine algebraic varieties. The Zariski topology. Morphisms. Dimension. Hilbert basis theorem. Hilbert Nullstellensatz. The co-ordinate ring. The spectrum of a ring

Unit 2

Projective space. Projective varieties. Projective closure. Morphisms of projective varieties. Automorphisms of projective space. Quasi-projective varieties. A basis for the Zariski topology. Regular functions.

Unit 3

Classical constructions.

Unit 4

Smoothness. Bertini's theorem. The Gauss mapping.

Text book: (An invitation to) Algebraic Geometry – K. E. Smith, L. Kahnpaa, P.Kekalainen and W. Treves, Springer (2000) relevant portions of Chapters 1 to 7.

References:

1. Undergraduate Commutative Algebra – Miles Reid, Cambridge Univ. Press. (1995)
2. Introduction to Commutaive Algebra – M. F. Atiyah & I. G. MacDonald – Addison-Wesley (1969)
3. Algebraic Geometry – Keith Kendig – Springer

4. Undergraduate Algebraic Geometry – Miles Reid, Cambridge Univ. Press (1988)
5. Hartshorne, R. – Algebraic Geometry, Springer-Verlag (1977)
6.Shafarevich I. R. – Basic Algebraic Geometry, Springer-Verlag (1974).

2 PROJECTIVE GEOMETRY

Unit 1

What is Projective Geometry? Projectivities. Perspectivities. Triangles and quadrangles: axioms and simple consequences. Perspective triangles. Quadrangular sets. Harmonic sets. The principle of duality. The Desargues' configuration. The invariance of the harmonic relation. .Trilinear polarity. Harmonic nets..

Unit 2

The fundamental theorem and Pappus' theorem: The axis of a projectivity. Pappus and Desargues. One-dimensional projectivities. Superposed ranges. Parabolic projectivities. Involutions and hyperbolic involutions. two dimensional projectivities. Projective, perspective and involutory collineations. Projective correlations.

Unit 3

Polariies: Conjugate points and conjugate lines. polar triangles. The use of self-polar pentagon. A self-conjugate quadrilateral. Product of two polarities. Self-polarity of the Desargues configuration. The conic. The polarity induced by a conic. Projectivity related pencils. Steiner's definition for a conic. The conic touching five given lines. The conic through five given points. Conics through four given points. Degenerate conics.

Unit 4

A finite projective plane. S combinatorial scheme for PG(2,5). Involution. Collineation and correlation conic. Is the circle a conic? Affine space. the language of pencils. The plane at inifinity. Euclidean space.

Text Book:

H.S.M. Coxeter – Projective Geometry (2nd edn.)—Univ. of Toronto Press (1974) . (The whole book).

References:

1. Struik D. J.– Lectures on Analytic and Projective Geometry – Addison-Wesley, 1953
2. Coxeter H. S. M.- The real Projective Plane (1955)

3 ADVANCED COMPLEX ANALYSIS

Unit 1

Partial fractions. Infinite products. Canonical products. The Gamma function. Stirling's formula. Entire functions. Jensen's fomula. Hadamard's theorem (without proof) (Chapters 5, section 2)

Unit 2

Riemann mapping theorem. Boundary behaviour. Use of reflection principle. Analytic arcs. Conformal mapping of polygons. The Schwarz-Christoffel formula. Mapping on a rectangle. The triangle functions of Schwarz. Functions with men value property. Harnack's principle (Ch. 6 Sections 1,2,3)

Unit 3

Subharmonic functions. Solutions. Solution of Dirichlet problem . Simply periodic functions. Doubly periodic functions. Unimodular transformations. Canonical basis. Generl properties of elliptic functions. (Ch. 6 Sections ,4 and Chapter 7 sections 1,2)

Unit 4

The Weierstrass ρ-function. The functions $\zeta(z)$ and $\sigma(z)$. The modular function $\lambda(\tau)$. Conformal mapping by $\lambda(\tau)$. Analytic continuation. Germs and sheaves. Sections and Riemann surfaces. Analytic continuations along arcs. Monodromy theorem.. (Ch. 7, section 3 and Chapter 8 section up to and including 1.6)

Text Book: Ahlfors L. V. – Complex Analysis (3rd edition) McGraw Hill International.

References:

1. Conway J. B. – Functions of one complex variable – Narosa (2002)
2. Lang S.– Complex Analysis – Springer (3rd edn.) (1995)
3. Karunakaran, V. – Complex Analysis – Alpha Science International Ltd. (2nd edn.) 2005.

4 ANALYTICAL MECHANICS

Unit 1

Equation of Motion, Conservation Laws

Unit 2

Integration of the equations of motion. Free Oscillation in one dimension

Unit 3

Motion of a rigid body

Unit 4

Hamilton's equations, Routhian, Poisson brackots, Action as a function of the coordinates, Maupertius principles

Text book: L.D. Landau and E.M. Lifshitz-Mechanics (3rd Edition) Porgamon 1976 [Relevant sections of Chapters 1, 2, 3, 5, 21, 6, 7.40 – 7.50]

References:

1.Herbert Goldstein (1980), Classical Mechanics, 2nd Ed., Narosa)
2.N.C. Rana and P.S. Joag (1991), Classical Mechanics, Tata Mc Graw Hill
3. K.C. Gupta (1988), Classical Mechanics of Particles and Rigid Bodies, Wiley Eastern
4.F.R. Gantmacher (1975), Analytical Mechanics, MIR Publishers.

5 STOCHASTIC PROCESSES

Unit 1

A brief description of Markov Process, Renewal Process, Stationary Process. Markov Chains: n-step transition probability matrix, classification of states, canonical representation of transition probability matrix, finite Markov chains with transient states. Irreducible Markov Chains with ergodic states: Transient and limiting behaviour.

Unit 2

First passage and related results. Branching Processes and Markov chains of order larger than 1, Lumpable Markov Chains, Reversed Markov Chains.

Unit 3

Applied Markov Chains: Queuing Models, Inventory Systems, Storage models, Industrial Mobility of Labour,, Educational Advancement, Human Resource Management, Term Structure, Income determination under uncertainty, Markov decision process. Markov Processes: Poisson and Pure birth processes, Pure death processes, Birth and death processes, Limiting distributions.

Unit 4

Markovian Networks. Applied Markov Processes: Queueing models, Machine interference problem, Queueing networks, Flexible manufacturing systems, Inventory systems, Reliability models, Markovian Combat models, Stochastic models for social networks; Recovery, relapse and death due to disease.

Text book: U.N. Bhat and Gregory Miller: Elements of Applied Stochastic Processes, Wiley Interscience, 2002 (Chs. 1, 2, 3, 4, 6, 7, 9.1-9.4, 9.9 and 10)

References:

1. Karlin and Taylor: A First Course in Stochastic Processes, Academic Press, 1975
2. E.Parzen: Stochastic Processes, Wiley 1968.
3. J.Medhi: Introduction to Stochastic Processes, New Age International Publishers, 1994, Reprint 1999.

6 FLUID MECHANICS

Unit 1

Introduction. The continuum hypothesis, volume forces and surface forces, transportation phenomenal, real fluids and ideal fluids. Poperties of gases and liquids. Pressure and thrust. Kinematics of the flow. Differentiation following the motion of the fluid. Velocity of a fluid at a point. Lagrangian and Eulerian description. Acceleration. Equation of continuity, stream lines and path lines.

Unit 2

Equation of motion of an ideal fluid. Euler's equation of motion. Bernoulli's equation, surface condition, velocity potential. Irrotational and rotational motion.

Unit 3

Particular method and applications. Motion in two dimensions, stream function, complex potential. Sources and sinks, Doublets. Images, conformal transformation and its application in Fluid Mechanics. Elements of vortex motion. vorticity and related theorems, line vortices, vortex street. Karman vortex street. Helmholtz theorems on vorticity.

Unit 4

General theory of irrotational motion. Flow and circulation, constancy of circulation. Minimum kinetic energy. Motion of cylinders. Forces on a cylinder, Theorem of Blasius, Theorems of Kutta and Joukowski. Axi-symmetric flows. Stokes stream function, motion of sphere.

.

Text Books: 1.Frank Chorlton – A Text Book of Fluid Dynamics – ELBS and Van Nostrand (1967) Chs. 2-5 and Ch.8

2. R. von Mises and K. O. Friedricks – Fluid Dynamics – Springer Verlag (1971)

Reference Books:

1) Davies – Modern Developments in Fluid Mechanics vol I & II – Van Nostrand
2) W.H.Besant and A.R.Ramsey – A Treatise on Hydromechanics Part II – ELBS
3) L.M.Milne Thomson – Theoretical Hydrodynamics Mac Millan (1962)

7 ALGEBRAIC TOPOLOGY

Unit 1

Geometric complexes and polyhedra. Orientation of geometric complexes.

Unit 2

Simplicial homology groups. Structure of homology groups. The Euler-Poincare theorem. Pseudomanifolds and the homology groups of Sn.

Unit 3

Simplicial approximation. Induced homomorphisms on homology groups. Brouwer fixed point theorem and related results.

Unit 4

The fundamental groups. Examples. The relation between H1(K) and π1(|K|).

Text Book: Fred H. Croom – Basic Concepts of Algebraic Topology – Springer Verlag (1978)

References:

1) Maunder – Algebraic Topology – Van Nostrand-Reinhold (1970)
2) Munkres J.R. – Topology, A First Course – Prentice Hall (1975)

8 NUMERICAL ANALYSIS AND COMPUTING

Unit 1

1 Principles of Numerical Calculations

1.1 Common Ideas and Concepts, Fixed-Point Iteration, Newton's Method, Linearization and Extrapolation, Finite Difference Approximations,

1.2 Some Numerical Algorithms, Solving a Quadratic Equation, Recurrence Relations. Divide and Conquer Strategy.

1.3 Matrix Computations, Matrix Multiplication, Solving Linear Systems by LU Factorization, Sparse Matrices and Iterative Methods, Software for Matrix Computations.

1.4 The Linear Least Squares Problem, Basic Concepts in Probability and Statistics, Characterization of Least Squares Solutions, The Singular Value Decomposition, The Numerical Rank of a Matrix

1.5 Numerical Solution of Differential Equations, Euler's Method , Introductory Example, Second Order Accurate Methods.

Unit 2

2. How to Obtain and Estimate Accuracy
2.1 Basic Concepts in Error Estimation, Sources of Error, Absolute and Relative Errors, Rounding and Chopping.

2.2 Computer Number Systems, The Position System, Fixed- and Floating-Point Representation, IEEE Floating-Point Standard.,Elementary Functions, Multiple Precision

2.3 Accuracy and Rounding Errors, Floating-Point Arithmetic, Basic Rounding Error Results, Statistical Models for Rounding Errors, Avoiding Overflow and Cancellation.

Unit 3

3. Interpolation and Approximation

3.1 The Interpolation Problem, Bases for Polynomial Interpolation, Conditioning of Polynomial

3.2 Interpolation Formulas and Algorithms, Newton's Interpolation ,Inverse Interpolation, Barycentric Lagrange Interpolation, Iterative Linear Interpolation, Fast Algorithms for Vandermonde Systems, The Runge Phenomenon

3.3 Generalizations and Applications, Hermite Interpolation, Complex Analysis in Polynomial Interpolation, Rational Interpolation, Multidimensional Interpolation.

3.4 Piecewise Polynomial Interpolation, Bernštein Polynomials and Bézier Curves, Spline Functions, The B-Spline Basis, Least Squares Splines Approximation. The Fast Fourier Transform. The FFT Algorithm.

Unit 4

4. Numerical Integration

4.1 Interpolatory Quadrature Rules ,Treating Singularities, Classical Formulas, Super-convergence of the Trapezoidal Rule, Higher-Order Newton–Cotes' Formulas

4.2 Integration by Extrapolation , The Euler–Maclaurin Formula, Romberg's Method, Oscillating Integrands Adaptive Quadrature

Text Books:

1. Numerical Analysis in Scientific Computing (Vol.1) Germund Dahlquist, Cambridge University press)
2. Numerical Recipes in C – The art of scientific computing (3rd edn.)(2007) – William Press (also available on internet)

Reference Books:

1. Applied Numerical Analysis using MATLAB. (2nd Edn) Laurene Fausett (Pearson)

2. Numerical Analysis: Mathematics of Scientific Computing, David Kincaid, Chency et.al. , Cengage Learning (Pub), 3rd Edn
3. Deuflhard P. & A. Hofmann – Numerical Analysis in Modern Scientific Computing – Springer (2002).

9 GRAPH THEORY

Unit 1

Basic results. Directed graphs. (Chapters I and II of the Text)

Unit 2

Connectivity. Trees (Chs. III and IV)

Unit 3

Independent sets and matchings. Eulerian and Hamiltonian graphs. (Chs. V and VI)

Unit 4

Graph colourings (Ch. VII)

Text Book: R. Balakrishnan, K. Ranganathan – A Text Book of Graph Theory –Springer (2000)

References:

1. C. Berge – Graphs and Hypergraphs – North Holland (1973)
2. J. A. Bondy and V. S. R. Murty – Graph Theory with Applications, Mac Millan 1976
3. F. Harary – Graph Theory – Addison Wesley, Reading Mass. (1969)
4. K. R. Parthasarathy – Basic Graph Theory – Tata McGraw Hill (1994)

10 FRACTAL GEOMETRY

Unit 1

Mathematical background. Hausdorff measure and dimension (Chapters 1 and 2 except sections 2.4 and 2.5)

Unit 2

Alternate definitions of dimension (Ch. 3)

Unit 3

Local structure of fractals. Projections of fractals (Chs. 5 and 6)

Unit 4

Products of fractals. Intersection of fractals (Chs. 7 and 8)

Text Book:

Frctal Geometry, Mathematical Foundations and Applications by Kenneth Falconer, John Wiley (1990)

Reference:

Mandelbrot B. B. – The Fractal Geometry of Nature – Freeman (1982).

11 CODING THEORY

Unit 1

Introduction to Coding Theory. Correcting and detecting error patterns. Weight and distance. MLD and its reliability. Error-detecting codes. Error correcting codes. Linear codes (Chapter 1 of the Text and sections 2.1 to 2.5 of chapter 2 of the text)

Unit 2

Generating matrices and encoding. Parity check matrices. Equivalent codes. MLD for linear codes. Reliability of IMLD for linear codes , Some bounds for codes,Perfect codes , Hamming codes. Extended codes extended Golay code and Decoding of extended Golay code (Sections 2.6 to 2.12 of Chapter 2 and sections 3.1 to 3.6 of chapter 3)

Unit 3

The Golay code, Reed-Muller codes, Fast decoding of RM(1,m) , Cyclic linear codes. Generating and parity check matrices for cyclic codes. Finding cyclic codes. Dual cyclic codes (Sections 3.7 to 3.9 of Chapter .3 and Chapter 4 complete)

Unit 4

. BCH codes. Decoding 2-error-correcting BCH code. Reed-Solomon codes. Decoding (Chapter 5 complete and sections 6.1, 6.2 and 6.3 of chapter 6)

Text Book:

Coding Theory and Cryptography The Essentials (2nd edition) – D. R. Hankerson, D. G. Hoffman, D. A. Leonard, C. C. Lindner, K. T. Phelps, C. A. Rodger and J. R. Wall – Marcel Dekker (2000)

Reference Books:

1.J. H. van Lint – Introduction to Coding Theory – Springer Verlag (1982)
2.E. R. Berlekamp – Algebraic Coding Theory – McGraw Hill (1968)

12 CRYPTOGRAPHY

Unit 1

Classical cryptography. Some simple cryptosystems. Cryptanalysis (Chapter 1 of the Text)

Unit 2

Shannon's theory (Ch. 2)

Unit 3

Block ciphers and the advanced encryption standard (Ch. 3)

Unit 4

Cryptographic hash function. (Ch. 4)

Text book: Cryptography, Theory and Practice – Douglas R. Stinson – Chapman & Hall (2002)

References:

1. N. Koblitz – A Course in Number Theory and Cryptography (2nd edition) Springer Verlag (1994)
2. D.R.Hankerson etc. – Coding Theory and Cryptography The Essentials – Marcel Dekker

13 NUMBER THEORY

Unit 1

Basic representation theorem. The fundamental theorem of arithmetic; combinatorial and computational number theory: Permutations and combinations, Fermat's little theorem, Wilson's theorem, Generating functions; Fundamentals of congruences- Residue systems, Riffling; Solving congruences- Linear congruences, Chinese remainder theorem, Polynomial congruences.

Unit 2

. Arithmetic functions- combinatorial study of phi (n), Formulae for d (n) and sigma (n), multivariate arithmetic functions, Mobius inversion formula; Primitive roots- Properties of reduced residue systems, Primitive roots modulo p; Prime numbers- Elementary properties of Pi (x), Tchebychev's theorem.

Unit 3

. Quadratic congruences: Quadratic residues- Euler's criterion, Legendre symbol, Quadratic reciprocity law; Distribution of Quadratic residues- consecutive residues and nonresidues, Consecutive ttriples of quardratic residues.

Unit 4

Additivity: Sums of squares- sums of two squares, Sums of four squares; Elementary partition theory- Graphical representation, Euler's partition theorem, Searching for partition identities;

Partition generating functions- Infinite products as generating functions, Identities between infinite series and products.

Text Book: George E Andrews: Number Theory, Dover Publications (1971) Chs.3 to 15.

References

1. Andre Weil-Basic Number Theory (3rd edn.) Springer-Verlag (1974)
2. Grosswald, E.-Introduction to Number Theory Brikhauser (2nd edition) 1984.

14. ANALYTIC NUMBER THEORY

Unit 1

The Fundamental theorem of Arithmetic, Arithmetical functions and Dirichlet multiplications

Unit 2

Averages of arithmetical functions. Some elementary theorems on the distribution of prime numbers.

Unit 3

Congruences. Finite abelian groups and their characters.

Unit 4

Dirichlet's theorem on primes in arithmetic progressions. Periodic Arithmetical Functions and gauss sums

Text Book: Tom M. Apostol- Introduction to Analytic Number Theory (Springer International Edn. 1998) Relevant portions from Chapters 1-10

References

1. G.H.Hardy & Wright Introduction to Theory of Numbers (Oxford) 1985
2. H.Davenport- The Higher Arithmetic (Cambridge) (6th edn.) 1992.

15 ALGEBRAIC NUMBER THEORY

Unit 1

Algebraic background, Symmetric Polynomials, modules, Free abelian groups, Algebraic numbers, Conjugates and discriminants, algebraic integers, integral basis, norms and traces, Rings of integers.(Sections 1.4-1.6, 2.1-2.6 of the text book)

Unit 2

Quadratic fields. Cyclotomic fields. Factorization into irreducible: Historical back ground. Trivial factorization int irreducible (Sections 3.1, 3.2, 4.1-4.3 of the text book)

Unit 3

Examples of non-unique factorization into irreducible. Prime factorization, Euclidean domains, Euclidean quadratic fields. Congruences of unique factorization Ramanajuan-Nagell theorem. (Sections 4.4-4.9 of the text book)

Unit 4

Ideals, Historical background, Prime factorization of ideals. The norm of an ideal. Non-unique factorization in cyclotomic fields. Lattices. The quotient torus. (Sections 5.1-5.4, 6.1, 6.2 of the text book)

Text Book:1.N.Stewart & D.O.Tall-Algebraic Number Theory (2nd edn.) Chapman & Hall (1987)

References:

1. P.Samuel- Theory of Algebraic numbers-Herman Paris Houghton Mifflin (1975)
2. S Lang-Algebraic Number Theory-Addison Wesley (1970)

16. FUZZY MATHEMATICS

Unit 1

From classical (crisp) sets to fuzzy sets: characteristics and significance of the paradigm shift. Additional properties of α-cuts. Representation of fuzzy sets. Extension principle for fuzzy sets. (Chs. 1 & 2 of the Text Book)

Unit 2

Operations on fuzzy sets. Types of operations. Fuzzy complements. t-norms, t-conorms. Combinations of operations. Aggregate operations. , Fuzzy numbers Arithmetic operations on intervals. Arithmetic operations on fuzzy numbers. Lattice of fuzzy numbers (Sections 3.1 to 3.4 of Ch. 3 of the Text and sections 4.1 to 4.5)

Unit 3

Crisp and fuzzy relations, projections and cylindric extensions, binary fuzzy relations, binary relations on a single set,Fuzzy equivalence relations , Compatibility and ordering relations. (sections 5.1 to5.6 of chapter 5 of text 5)

Unit 4

Fuzzy morphisms. sup-i, inf- ωicompositions of fuzzy relations. Fuzzy logic. Fuzzy propositions. Fuzzy quantifiers. Linguistic hedges. Inference from conditional, conditional and qualified and quantified propositions (Sections 5.8 to 5.10 of Ch. 5 of the Text, and Ch. 8 of the Text)

Text Book: Fuzzy sets and Fuzzy logic Theory and Applications – G. J. Klir & Bo Yuan – PHI (1995)

References :

1. Zimmermann H. J. – Fuzzy Set Theory and its Applications, Kluwer (1985)
2. Zimmermann H. J. – Fuzzy Sets, Decision Making and Expert Systems, Kluwer (1987)
3. Dubois D. & H. Prade – Fuzzy Sets and Systems: Theory and Applications – Academic Press (1980)

17 OPERATIONS RESEARCH

Unit 1

Linear programming in two-dimensional spaces. General LP problem. Feasible, basic and optimal solutions, simplex method, simplex tableau, finding the first basic feasible solution, degeneracy, simplex multipliers. (Chapter 3 Sections 1-15).

Unit 2

The revised simplex method. Duality in LP problems, Duality theorems, Applications of duality, Dual simplex method, summary of simplex methods, Applications of LP. (Chapter 3. Sections 16-22)

Unit 3

Transportation and Assignment problems (Chapter 4)

Unit 4

Integer programming. Theory of games (Chapters 6 and 12)

Text Book: K. V. Mital and C. Mohan – Optimisation Methods in Operations Research and Systems Analysis (3rd edition) -New Age International (1996).

Reference Books:

1. Wagner – Operations Research, Prentice Hall India
2. A. Ravindran, Don T. Philips, James Solberg – Operations Research, Principles and Practice – John Wiley (3rd edition)
3. G. Hadley – Linear Programming – Addison Wesley
4. Kanti Swarup, P.K.gupta, Man Mohan – Operations Research – S. Chand & Co.

Model Question Paper (2016 Admission)
First Semester MSc.Mathematics
CCSS
Kannur University
MAT C 101 : ALGEBRA I

Time: 3 Hours. **Max. Marks** : 60

(Each question carries 15 marks .Answer any **ONE** question from each unit)

UNIT I

1. a) Define simple group and show that M is a maximal normal subgroup of G if and only if G/M is simple. (5)
   b) Show that the group $\mathbb{Z}_m \times \mathbb{Z}_n$ is cyclic and is isomorphic to $\mathbb{Z}_{mn}$ if and only if m and n are relatively prime. (5)
   c) Find ker $\emptyset$ for $\emptyset: Z \times Z \rightarrow Z$ where $\emptyset(1,0) = 3$ and $\emptyset(0,1) = -5$. (2)
   d) Prove that a factor group of a cyclic group is cyclic. (3)
2. a) Let G be a group and C be the commutator subgroup of G.Then prove that C is a normal subgroup of G.Also prove that , if N is a normal subgroup of G then G/N is abelian iff C≤N. (5)
   b) Let H be a subgroup of a group G.Then prove that left coset multiplication is well defined by the equation (aH)(bH)=(ab)H iff H is a normal subgroup of G. (4)
   c) Prove that A_4 cannot contain a subgroup of order 6. (3)
   d) Prove that $\gamma: G \rightarrow G/H$ given by $\gamma(x) = xH$ is a homomorphism with kernel H. (3)

UNIT II

3. a) State and prove Cauchy's theorem. (5)
   b) Prove that for a prime number p, every group G of order p^2 is abelian (4)
   c) Compute the fixed sets X_σ for each $\in D_4$. (3)
   d) State and prove second Sylow theorem (3)

4. a) Derive Burnside's formula. (5)
   b) Prove that the center of a finite nontrivial p-group G is nontrivial. (4)

c)State and prove first Sylow theorem. (6)

UNIT III

5. a) Show that any integral domain D can be enlarged to a field F. (8)

b) Let G be a finitely generated abelian group with generating set $\{a_1, a_2, \dots, a_n\}$ .Let $\emptyset: Z \times Z \times \dots \times Z \rightarrow G$ be defined by $\emptyset(h_1, h_2, \dots, h_n) = h_1 a_1 + h_2 a_2 + \cdots + h_n a_n$.Then show that $\emptyset$ is a homomorphism onto G. (4)

c) Define free group generated by the set A (3)

6. a) State and prove the Evaluation homomorphisms for field theory (6)

b) Let F be a field of quotients of D and let L be any field containing D.Then show that (6)

there exist a map $\psi: F \rightarrow L$ that gives an isomorphism of F with a subfield of L such that $\psi(a) = a$ for $\in D$.

(c) Define the rank of a group G. Find the rank of the group $\mathbb{Z} \times \mathbb{Z}$. (3)

UNIT IV

7. a) State and prove division algorithm in F[x.] (6)

b) Let F be a field and f(x),g(x)∈ F[x] . Show that f(x) divides g(x) iff g(x)∈⟨f(x)⟩. (3)

c) Demonstrate that $x^3 + 3x^2 - 8$ is irreducible over Q. (4)

d) Show that if R is a ring with unity and N is an ideal of R containing a unit then N=R. (2)

8. a) State and prove Eisenstein criterion theorem. (5)

b) Check irreducibility of $8x^3 + 6x^2 - 9x + 24$. (3)

c) If F is a field then show that every ideal in F[x] is principal. (4)

d) State and prove factor theorem. (3)

Model Question Paper(2016 Admission)
First Semester M.ScMathematics
(CCSS)
Kannur University
MAT C 102 **Linear Algebra**

(Each question carries 15 marks .Answer any **ONE** question from each unit)

Time: 3Hours. Max.Mark:60

Unit – I

1. a) Let V and W be vector spaces over the field F and let T be a linear transformation from V into W. Prove that if V is finite dimensional, then rank(T) + nullity T = dim V
   b) Find a basis for the space $L(R^2, R^3)$ over R.
   c) Show that a linear transformation T is if and only if T carries linearly independent sets onto linearly independent sets

2. a) Let B and B' be two ordered bases for an n dimensional vector space V over the field F and T be a linear operator on V. Then prove that there exist an invertible nxn matrix P over F such That $[T]_{B'} = P^{-1}[T]_B P$

   b) Find range an null space of the linear operator T(a,b,c)=(a+b,2c,0) on R^3.
   c)Show that every n dimensional vector space over the field F is isomorphic to F^n

Unit- II

3.a) If V is a finite dimensional vector space over the field ,and W is a subspace of V then prove that $\dim W + \dim W^0 = \dim V$

b)If S is any subset of a finite dimensional vector space then prove that($S^0)^0$ is the subspace spanned by S

c)If A is any mXn matrix over the field F.Then show that the row rank of A is equal to the column rank of A

4.a)Show that similar matrices have the same Characteristic polynomial

.

b) Define diagonalizable operator. Let T be a linear operator on R^3 which is represented in the standard basis by the matrix $A=\begin{matrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{matrix}$ Show that T is diagonalizable

c) If $T\alpha = c\alpha$ and f is any polynomial show that $f(T)\alpha = f(c)\alpha$

Unit III

5. a) State Prove the Cayley-Hamilton theorem.

b) Define minimal polynomial. Show hat for a linear operator T ,the characteristic and minimal polynomial will have the same roots.

c)Find the minimal polynomial for the operator on R^3 for which the matrix relative to the

standard basis is $\begin{matrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{matrix}$

6. a) State and prove a necessary and sufficient condition for a linear operator on a finite dimensional vector space to be diagonalizable.

   b) Find an invertible matrix P such that P^{-1} AP is a diagonal matrix, where

   $A = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$.

   c) If V is finite dimensional vector space and W_1 is any subspace of V prove that there is a subspace W_2 of V such that $W_1 + W_2$

Unit IV

7. a) State and prove Primary decomposition theorem.

   b) if $T(x_1,x_2)= (2x_1, x_2, -x_1)$ find a diagonalizable operator D and a nilpotent operator N on R^2 such that T=D+N

8. a) Define an inner product space. Prove that an orthogonal set of non-zero vector in an inner product space is linearly independent.

   b) If W is a finite dimensional subspace of an inner product space V then show that $V=W\oplus W^{\perp}$ where $W^{\perp}$ is the orthogonal complement of W in V.

   c) Describe Gram – Scmidt Orthogonaization process

Model Question Paper(2016 Admission)

First Semester M.Sc. Mathematics Examination

CCSS

Kannur University

MAT C 103 – Differential Equations I

[Each question carries 15 marks, students have to answer one question from the two questions given from each unit]

Unit I

Time: 3 Hrs Max Marks 60

1. a) State and prove Picard's existence and uniqueness theorem for first order differential equations -5-

b) Solve the initial value problem $y' = x + y$, $y(0) = 1$, taking $y_0(x)=1$ .Apply Picard's method to calculate $y_n(x)$. Does y_n converges to the actual solution? -4-

c) Solve the initial value problem $y' = y^2$, $y(0)=1$ -3-

d) If $q(x) < 0$ and $u(x)$ is a nontrivial solution of $u''(x) + q(x)\,u = 0$ show that $u(x)$ has at most one zero -3-

2. a) Define Lipschitz condition -4-

Show that $f(x, y) = y^{\frac{1}{2}}$

b) Does not satisfy a Lipschitz condition on the rectangle $|x| \leq 1$ and $0 \leq y \leq 1$ -2-

c) Does satisfy a Lipschitz condition on the rectangle $|x| \leq 1$ and $c \leq y \leq d$ -6-

d) Let $y(x)$ be a nontrivial solution of $y'' + q(x)\,y = 0$ on a [a b] show that $y(x)$ has at most a finite number of zeros in this interval -3-

Unit II

3.a) Express $sin^{-1}x$ in the form of a power series $\sum a_n x^n$ by solving $y' = \frac{1}{\sqrt{1-x^2}}$ in two ways. Deduce that $\frac{\pi}{6} = \frac{1}{2} + \frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{2^3} + \frac{1}{2}\cdot\frac{3}{4}\cdot\frac{1}{5}\cdot\frac{1}{2^5} + \cdots$ -4-

b) Solve Legendre's equation $(1- x^2)\,y'' - 2xy' + p(p+1)\,y = 0$ where p is a constant , ' 0 ' being an ordinary point -3-

c) Find the general solution of $(1+x^2)y''+2xy' -2y = 0$ in terms of power series in x. Can you express this solution by means of elementary function -4-

d) Verify that the equation $y''+ y' -x\,y = 0$ has a three term recursion formula and find it's solutions $y_1(x)$ and $y_2(x)$ such that $y_1(0) = 1$, $y_1'(0) = 0$, $y_2(0) = 0$, $y_2'(0) = 1$ -4-

4.a) Determine the nature of the point $x = \infty$ for Legendre's equation

$(1-x^2)\,y'' - 2xy' + p(p + 1)\,y = 0$ -3-

b) Using Confluent hyper geometric function in Lagurre's equation

$x\,y'' + (1-x)\,y' + p\,y = 0$, p a constant .Show that the only solution bounded near the origin are constant multiples of $F(-p, 1, x)$ -4-

c) show that the equation $x^2y''-3xy' + (4x + 4) y = 0$ has only one Frobenius series solution -4-

d) Find the general solution of $4x^2y''-8x^2y'+ (4x^2+1)y =0$ -4-

Unit III

5. a)If the function f(x) has the Legendre series $f(x) = \sum_{n=0}^{\infty} a_n\, p_n(x)$ show that $a_n = \frac{2n+1}{2}\int_{-1}^{1} f(x)\, p_n(x)dx$, where $p_n(x)$ is the Legendre polynomial -3-

b) Find the first three terms of the Legendre series of $f(x) = 0$ if $-1 \le x < 0$

$= x$ if $0 \le x \le 1$ -4-

c) If W(t) is the Wronskien of two solutions $\begin{cases} x = x_1(t) \\ y = y_1(t) \end{cases}$ and $\begin{cases} x = x_2(t) \\ y = y_2(t) \end{cases}$ of the system $\begin{cases} \frac{dx}{dt} = a_1(t)x + b_1(t)y \\ \frac{dy}{dt} = a_2(t)x + b_2(t)y \end{cases}$ on [a,b] then show that W(t) is either identically zero or nowhere zero -4-

d) Find the general solution of $\begin{cases} \frac{dx}{dt} = x + y \\ \frac{dy}{dt} = 4x - 2y \end{cases}$ -4-

6. a)If λ_m *and* λ_n are distinct positive zeros of the Bessel function $J_p(x)$ .Show that

$\int_0^1 x\, J_p(\lambda_m x) J_p(\lambda_n x)dx = 0$ if $m \neq n$

$= \frac{1}{2} J_{p+1}(\lambda_n)^2$ if m = n -4-

b) If $f(x) = x^p$ for $0 \le x < 1$ show that its Bessel series in the functions $J_p(\lambda_n x)$ where $\boldsymbol{\lambda}_n$ are positive zeros of Jp(x) is $x^p = \sum_{n=1}^{\infty} \frac{2}{\lambda_n J_{p+1}(\lambda_n)} J_p(\lambda_n x)$ -4-

. c) Show that if the two solutions of the homogeneous system are linearly independent on [a,b], then $\begin{cases} x = c_1x_1(t) + c_2x_2(t) \\ y = c_1y_1(t) + c_2y_2(t) \end{cases}$ is a general solution of the homogeneous system $\begin{cases} \frac{dx}{dt} = a_1(t)x + b_1(t)y \\ \frac{dy}{dt} = a_2(t)x + b_2(t)y \end{cases}$ on [a,b] -4-

d) Find the general solution of $\begin{cases} \frac{dx}{dt} = 4x - 2y \\ \frac{dy}{dt} = 5x + 2y \end{cases}$ -3-

Unit IV

7. a) Determine the nature and stability of the critical point (0 , 0) of the linear autonomous system $\begin{cases} \frac{dx}{dt} = 4x - 3y \\ \frac{dy}{dt} = 8x - 6y \end{cases}$ -4-

b) Show that (0,0) is an asymptotically stable critical point for the system $\begin{cases} \frac{dx}{dt} = -2x + xy^3 \\ \frac{dy}{dt} = -x^2y^2 - y^3 \end{cases}$ -4-

c) Find the critical point and solve the differential equation to find the path of $\begin{cases} \frac{dx}{dt} = y(x^2 + 1) \\ \frac{dy}{dt} = 2xy^2 \end{cases}$ -4-

d) Determine the nature and stability properties of the critical point (0,0) for the system i) $\begin{cases} \frac{dx}{dt} = -x - 2y \\ \frac{dy}{dt} = 4x - 5y \end{cases}$ ii) $\begin{cases} \frac{dx}{dt} = 2x \\ \frac{dy}{dt} = 3y \end{cases}$ -3-

8. a) Show that (0,0) is an asymptotically stable critical point for the system $\begin{cases} \frac{dx}{dt} = -y - x^3 \\ \frac{dy}{dt} = x - y^3 \end{cases}$ but is an unstable critical point of $\begin{cases} \frac{dx}{dt} = -y + x^3 \\ \frac{dy}{dt} = x + y^3 \end{cases}$ -5-

b)Determine the definiteness of following functions

i)$x^2 - xy - y^2$ ii)$-2x^2 + 3xy - y^2$ -2-

c) Consider the equation of motion of a mass attached to a string $m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0$ Where c$\geq$ 0 is a constant representing the viscosity of the medium through which the mass moves and k>0 is the string constant. Discuss about the stability of the critical point -4-

d) Determine the nature and stability of the critical point (0,0) for the system $\begin{cases} \frac{dx}{dt} = 4x - 2y \\ \frac{dy}{dt} = 5x + 2y \end{cases}$ -4-

Model Question paper (2016 Admission)

First Semester MSc. Mathematics Examination

CCSS

Kannur University

MAT C 104: REAL ANALYSIS

Time: 3 hrs **Max. Marks**: 60

(Each question carries 15 marks, students have to answer one question from the two questions given from each unit)

UNIT I

1. a) Show that every infinite subset of a countable set A is countable. (5)
b) Show that a countable union of countable set is countable.
c) Let A be a countable set, and B_n be the set of all n-tuples $(a_1, a_{2,} \dots, a_n)$, where $a_k \in$ (5)
A $(k = 1,2, \dots, n)$,and the elements $a_1, a_{2,} \dots, a_n$ need not be distinct. Then B_n is countable.
(5)

2. a) Let A be the set of all sequences whose elements are the digits 0 and 1. Show that this set A is uncountable. (6)
b) Prove that a set E is open if and only if its complement is closed. (3)
c) Show that every bounded infinite subset of R^k has a limit point in R^k (3)
d) Let f be a continuous mapping of a compact metric space X into a metric space Y. Then show that f is uniformly continuous on X. (3)

UNIT II

3. a) If f and g are continuous real functions on $[a, b]$ which are differentiable in (a, b), then show that there is a point $x \in (a, b)$ at which $[f(b) - f(a)]g'(x) = [g(b) - g(a)f'(x)]$ (4)
b)What happens in the above theorem if you take $g(x) = x$. At that case is this theorem is applicable for complex valued function. Justify your answer. (3)
c) State and prove Taylor's theorem. (5)
d) Suppose f is a real differentiable function on $[a, b]$ and suppose $f'(a) < \lambda < f'(b)$. (3)
Then show that there is a point $x \in (a, b)$ such that $f'(x) = \lambda$.

4. a) State and prove L-Hospital's rule. (8)
b) Let f be defined for all real x, and suppose that $|f(x) - f(y)| \leq (x - y)^2$ for all real x and y. Prove that f is constant. (4)
c) Suppose f and g are defined on $[a, b]$ and are differentiable at a point $x \in [a, b]$. Then show that $f + g, fg, f/g$ are differentiable at x and
i) $(f + g)'(x) = f'(x) + g'(x)$; ii) $(fg)'(x) = f'(x)g(x) + f(x)g'(x)$;
iii) $\left(\frac{f}{g}\right)'(x) = \frac{g(x)f'(x) - g'(x)f(x)}{g^2(x)}$ (3)

UNIT III

5. a) Assume α increases monotonically and $\alpha' \in \mathcal{R}$ on $[a, b]$. Let f be a bounded real function on $[a, b]$, Then show that $f \in \mathcal{R}(\alpha)$ if and only if $f\alpha' \in \mathcal{R}$ and $\int_a^b f\, d\alpha = \int_a^b f(x)\alpha'(x)dx$. (6)
   b) Suppose $f \in \mathcal{R}(\alpha)$ on $[a, b]$, $m \leq f \leq M$, ϕ is continuous on $[m, M]$, and $h(x) = \phi(f(x))$ on $[a, b]$. Prove that $h \in \mathcal{R}(\alpha)$ on $[a, b]$. (4)
   c) Prove that if $f \in \mathcal{R}(\alpha)$ and $g \in \mathcal{R}(\alpha)$ on $[a, b]$, then $fg \in \mathcal{R}(\alpha)$. (2)
   d) If P^* is a refinement of P then show that $L(P, f, \alpha) \leq L(P^*, f, \alpha)$ and $U(P^*, f, \alpha) \leq U(P, f, \alpha)$. (3)
6. a) State and prove Riemann-Stieltjes criterion for integrability. (6)
   b) If γ',the derivative of γ, is continuous on $[a, b]$, then show that γ is rectifiable, and $\Lambda(\gamma) = \int_a^b |\gamma'(t)|\, dt$. (6)
   c) Prove that $f \in \mathcal{R}(\alpha)$ on $[a, b]$ if and only if for every $\varepsilon > 0$ there exists a partition P such that $(P, f, \alpha) - L(P, f, \alpha) < \varepsilon$. (3)

UNIT IV

7. a) State and prove Cauchy criterion for uniform convergence. (6)
   b) If K is compact, if $f_n \in \mathcal{C}(K)$ for $n = 1,2,3, \ldots$, and if $\{f_n\}$ is point wise bounded and equicontinuous on K, then show that (6)
      i) $\{f_n\}$ is uniformly bounded on K
      ii) $\{f_n\}$ contains a uniformly convergent subsequence.
   c) If $\{f_n\}$ is a sequence of continuous functions on E and if $f_n \to f$ uniformly on E then prove that f is continuous on E. (3)
8. a) State and prove Stone-Weierstrass theorem. (7)
   b) Prove that there exist a real continuous function on the real line which is nowhere differentiable. (5)
   c) Prove that if K is a compact metric space, if $f_n \in \mathcal{C}(K)$ for $n = 1,2,3, \ldots$, and if $\{f_n\}$ converges uniformly on K, then $\{f_n\}$ is equicontinuous on K. (3)

Model Question Paper(2016 Admission)

First Semester M.Sc. Mathematics Examination

(Credit and Semester System)

Kannur University

MAT C 105 – Topology

[Each question carries 15 marks, students have to answer one question from the two questions given from each unit]

Unit I

Time: 3 Hrs | Max Marks 60

1. a) Define the product of sets $\{S_w\}_{w \in W}$ and show that this is a natural extension of the product of two element set $\{S_1, S_2\}$ -3-

   b) Show that in any topological space (S, χ), $A \subset S$ is closed if and only if $A = \bar{A}$ -4-

   c) Define basis for a topology, give an example. Also show that every metric space (S, ρ) is a topological space -4-

   d) If τ_1 and τ_2 be two collections of sets, then show that $(\cup_{S \in \tau_1} S) \cup (\cup_{S \in \tau_2} S) = \cup_{s \in (\tau_1 \cup \tau_2)} S$ and $(\cup_{S \in \tau_1} S) \cap (\cup_{S \in \tau_2} S) = \cap_{S \in (\tau_1 \cup \tau_2)} S$. -4-

2. a) Show that a topological space is connected if and only if it is not the union of two disjoint nonempty open sets. -4-

   b) Prove that a topological space is compact if and only if it has finite intersection property. -4-

   c) Show that every closed subset of a compact space is compact in its relative topology -3-

   d) If $A = \{a_1, a_2, \ldots\ldots\ldots, a_n\}$ is a finite set of n elements, then the set of all subsets of A has 2^n elements -4-

Unit II

3. a) Let S and T be topological spaces. Let $\boldsymbol{B}_S$ and $\boldsymbol{B}_T$ be bases for the topologies on S and T respectively. Then f: S → T is continuous if and only if for each $s \in S$ and each $v \in \boldsymbol{B}_T$ with $f(s) \in V$, there exists a $U \in \boldsymbol{B}_S$ such that $s \in U$ and $f(U) \subset V$. -4-

   b) let $f: S \rightarrow R^1$ is continuous . If S is compact, then show that f assumes its maximum and minimum on S. -4-

   c) If S and T are topological spaces, let f: S → T is continuous and surjective . If S is compact, then show that T is also compact -4-

   d) If S and T are topological spaces, let f: S → T is continuous and surjective . If S is connected, then show that T is also connected -3-

4. a) Define product topology on a class $\{ S_w \}_{w \in W}$ of topological spaces. Also show that $\mathcal{B} = \{ \cap_{w \in W_1} \prod_w^{-1}(\cup_w)$;W_1 is a finite subset of W and $\cup_w$ an open subset in S_w for each $w \in W_1$ }is a basis for the product topology. -6-

   b)Explain product topology with the help of the example I x I , where I=(0,1). -4-

   c)State Tychnonoff theorem. -1-

   d)Define product topology. Explain with the help of the example $\prod_{n \in J} In$ where $I_n = [0, 1/n]$ and J^+ is the set of positive integers -4-

Unit III

5. a)State and prove urysohn's lemma -5-
   b)Let A_0 and A_1 be disjoint closed subsets of a normal space S. Then prove that there exists a continuous function g: S→[a,b] with $g(A_0)$ = a and $g(A_1)$ = b -2-
. c) DefineT_0 , T_1, T_2 , T_3 and T_4 Space. Give one example for each -4-
   d) Show that every metric space is T_2 .Also show that S is a T_1space iff each point of s is closed as a subset of S -4-

6. a) Define a completely regular space and show that every completely regular space is regular -4-
   b) Show that every subset of a completely regular space is completely regular -4-
   c) Prove that every locally compact Hausdorff space is completely regular -3-
   d) Show that the 1-point compactification $\widetilde{S}$ of S is Hausdorff if and only if S is Hausdorff and locally compact -4-

Unit IV

7.a)Prove that there exists a continuous real valued function f ϵ C[0,1] such that f has a derivative at no point of [0,1] 5-
. b)Define a complete metric space -2-
   c) Define countably compact, sequentially compact and totally bounded metric spaces-4
   d) Prove that if S is countably compact then it is sequentially compact. Prove also that if S is sequentially compact iff it is complete and totally bounded -4-

8.a) Prove that if S is complete and totally bounded metric space then it is sequentially compact. Also prove that if S is complete totally bounded and sequentially compact then S is compact -6-
. b)Let (S, ρ) be a complete metric space. Suppose $\{U_n\}$ is a countable collection of open sets each of which is dense in S. Prove that $\bigcap_{n=1}^{\infty} U_n \neq \varphi$ -4-
   c) State and prove Baire category theorem -4-
. d)Define a complete metric space -2-